THE APPLICATION OF A COMPUTER-AIDED DESIGN
TECHNIQUE TO THE CONCEPTUAL DESIGN OF TURBOCHARGERS

Part A Centrifugal compressor design and turbine matching

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ABSTRACT

In many turbomachinery applications a compressor is directly driven by a turbine; for turbocharger applications a centrifugal compressor is usually adopted which is generally driven by a radial flow turbine, although mixed flow or axial flow turbines are occasionally required. A non-dimensional design procedure is developed to provide the basic dimensions and blade angles of centrifugal compressor impellers, whilst accounting for the turbine conditions as assessed through the matching requirements. The design of the turbine is then considered further in Part B.

The procedure can be applied for any desired compressor pressure ratio and target efficiency to develop an initial non-dimensional skeleton design. No other parameters are required from the initial specification and the design is developed non-dimensionally without recourse to empirical loss models and the associated uncertainties as the target efficiency must be specified. The procedure provides graphical information with respect to the impeller discharge conditions and inlet conditions from which the designer must select the most appropriate design. The screen graphics interface enables the designer to search across the design options; as this search is carried out numerical data are displayed and continuously up-dated to provide immediate information on which an informed assessment can be based.

In addition to the compressor design options which are provided the matching conditions for the drive turbine provide information, such as specific speed, non-dimensional mass flow rate and pressure ratio, relevant to the turbine design. Judgements with respect to the design options for the compressor can then be made with the consequences for the associated turbine design clearly in view.

The non-dimensional design can be translated into an absolute design through the specification of the required mass flow rate and the inlet stagnation pressure and temperature.

1 NOTATION

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<tbody>
<tr>
<td>M</td>
<td>Mach number</td>
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<tr>
<td>M_{st}</td>
<td>Relative Mach number</td>
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<td>M_{n}</td>
<td>Non-dimensional impeller speed, ( U_{n}/a_{n} )</td>
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<td>( \dot{m} )</td>
<td>Mass flow rate</td>
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<td>P</td>
<td>Pressure</td>
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<td>r</td>
<td>Radius</td>
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<tr>
<td>S_{c}</td>
<td>Power coefficient, ( W_{c}/nh_{u} )</td>
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<tr>
<td>S_{w1}</td>
<td>Inlet swirl coefficient, ( (C_{w1}/U_{1})/h_{w1} )</td>
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<tr>
<td>S_{w2}</td>
<td>Discharge swirl coefficient, ( (C_{w2}/U_{2})/h_{w2} )</td>
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<td>T</td>
<td>Temperature</td>
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<td>W_{c}</td>
<td>Compressor power</td>
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<td>\alpha</td>
<td>Absolute flow angle from the radial or axial direction, positive in direction of impeller rotation</td>
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<tr>
<td>\beta</td>
<td>Relative flow angle from the radial or axial direction, positive in direction of impeller rotation</td>
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<td>\theta</td>
<td>Non-dimensional mass flow rate, ( \dot{m}/(\rho_{a}a_{n}r_{h}^{2}) )</td>
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<tr>
<td>\lambda</td>
<td>Parasitic work parameter</td>
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<td>\nu</td>
<td>Impeller inducer hub-tip radius ratio</td>
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<tr>
<td>\eta_{c}</td>
<td>Compressor efficiency</td>
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<td>\eta_{i}</td>
<td>Impeller efficiency</td>
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<td>Inducer shroud tip position</td>
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<td>Station position at impeller inlet</td>
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<td>2</td>
<td>Station position at impeller discharge</td>
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<tr>
<td>3</td>
<td>Station position at stage discharge</td>
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Presented at the International Gas Turbine and Aeroengine Congress & Exhibition
Birmingham, UK — June 10-13, 1996
2 INTRODUCTION

The automotive turbocharger and many small gas turbine applications usually require a centrifugal compressor directly driven by a radial inflow turbine. The demand for fuel efficient engines with low exhaust emissions and which can operate over a wide speed range, has led to the requirement for turbochargers with a broad operating range and high pressure ratio whilst maintaining a high efficiency. The demand for increased compressor pressure ratio, whilst not wishing to increase the speed limitations of the turbine, has led to the consideration of mixed flow turbines as an alternative to the usual radial flow design, Chou and Gibs (1989) and Abidat et al (1992). Similarly any speed limitations impinge on the compressor design options and may restrict the degree of discharge blade backswep which can be adopted. This will lead to reduced operating range and consideration of the application of inlet swirl, Whitfield et al (1993). Whilst the design procedure presented here was developed with the turbocharger in view it is readily applicable to the design of gas turbine or industrial centrifugal compressors.

In order to provide a procedure which is widely applicable the number of empirical data and procedures used have been kept to a minimum. In addition to the aerodynamic requirements the designer must give due consideration to the need to provide a small overall frame size, low inertia components, generally durability and minimum costs. The precise weighting of these wide and often conflicting requirements must be addressed by the designer, and cannot be embodied within a computerised design procedure as the design criteria adopted will differ from one application to another, and with the experience and philosophy of the designer. As a consequence the design selection and assessment is left in the hands of the designer who has a full appreciation of all the design objectives and constraints.

For the design procedure to be able to accommodate all the aerodynamic and non-aerodynamic requirements and constraints encountered across the wide range of applications available it must start with a minimum of input requirements, and have no built-in constraints and empiricism which cannot be easily modified and assessed by the designer. The basic initial input requirement is for the specification of:

i) the compressor pressure ratio, and

ii) a target efficiency for the compressor and turbine.

The target efficiency is the primary empirical input requirement. This will probably be based on the designers knowledge of comparable machines and the need to balance efficiency requirements with other constraints, such as operating range and overall size. The uncertainty associated with the target efficiency can be readily assessed; eventually the performance of the proposed design must be predicted to ensure that the efficiency, and other performance parameters, can be achieved. The initial design procedure cannot take full account of the complex three dimensional separated flow which occurs in the impeller. It is, however, important to have these flow phenomena in mind and carry out the initial design with a view to minimizing the potential for flow separation and maximizing the efficiency within the bounds of any other design constraints. Assessment parameters such as relative velocity ratio, diffusion factor and specific speed are provided for this purpose.

For the compressor design the additional empirical parameters of impeller slip factor and efficiency are introduced within the program operation. The slip factor has a direct bearing on the magnitude of the discharge blade angle, and the impeller efficiency on the discharge blade height. Both of these parameters can be easily changed to assess the affect of any uncertainties involved.

The design procedure provides the designer with the ability to quickly assess the design options available for the complete turbocharger, whilst not being restricted in that assessment by any built-in empirical constraints of the programmed procedure. The overall dimensions of the turbine machines including the blade angles and the rotational speed are derived. The procedure allows for the design and assessment of:

i) radial or mixed flow compressors with and without inlet swirl, 

ii) radial and mixed flow turbines with and without discharge swirl, and

iii) the impact of the design of one component on the other through consideration of the turbine/compressor matching requirements.

The event driven programming procedure allows the user to select the options required and to re-assess designs as and when required. In essence the procedure presents, graphically, all the options available at inlet to and discharge from the rotating component, and allows the designer to make the necessary design choices. The graphical procedure adopted allows the designer to rapidly scan the design options available, and presents and continually up-dates essential parameters on which the design selection and assessment can be based.

The procedure is illustrated through the application to the design of a turbocharger compressor with a design pressure ratio of 3.6.

3 NON-DIMENSIONAL DESIGN

The non-dimensional design procedure is developed on the basis that the desired compressor pressure ratio together with a target efficiency is known from the outset. The further specification of the gas mass flow rate and inlet stagnation conditions is then sufficient to transform the non-dimensional design into absolute dimensions. Whilst the procedure adopted is non-dimensional the initial specification of a target efficiency is size related and must be assessed alongside the known constraints, such as a small and low inertia impeller. Rodgers (1991) derived a correlation for efficiency based on the four critical parameters of:

specifc speed, 

inducer tip relative Mach number, 

exit discharge Mach number, and 

impeller diameter.

The first three parameters are non-dimensional and can be derived and assessed as the design develops with a view to minimizing losses and maximizing efficiency. The impeller diameter, and rotational speed, is derived from the non-dimensional design through the specification of the mass flow rate and inlet stagnation conditions. If necessary the target efficiency can be modified during the design process; the correlations provided by Rodgers (1991) could be used for this purpose.

3.1 Initial specification

The initial requirement is for the specification of the compressor pressure ratio and target efficiency which are related through

$$\eta_c = \frac{(P_{RC})_1}{T_{32}} = \frac{(P_{RC})_T}{T_{11}} = \eta_{sc}$$

where the power coefficient, $S_{wc}$, is defined through the steady flow energy equation as

$$S_{wc} = \frac{W_c}{m_{t_i} C_{v_c} T_{11}} = \frac{T_{32}}{T_{11}} - 1$$

With the pressure ratio specified together with a target efficiency the power coefficient, $S_{wc}$, can be derived and the design is effectively developed for a known power coefficient. The corresponding power coefficient for the turbine design is derived through the power matching requirement, see below.

From the Euler turbomachinery equation the non-dimensional power coefficient can be developed as

$$\lambda = \frac{W_c}{m_{t_i}} = \frac{U_2 C_{s2}}{h_{s1}} - \frac{U_1 C_{s1}}{h_{s1}}$$

where $\lambda$ is a parasitic work parameter defined as

$$\lambda = \frac{1 - W_{pave}}{W_c}$$

Then

$$\lambda S_{wc} = S_{w1}$$

(3)
and discharge conditions respectively. Specification of a magnitude for \( S_{w2} \) is equivalent to designing with inlet prewhirl. For the usual design with zero inlet swirl \( S_{w1} = \lambda S_{w2} \); however, specification of positive inlet swirl, \( S_{w1} > 0 \), will require a corresponding increase in the magnitude of \( S_{w2} \). The magnitude of \( S_{w1} \) is derived through Eqn.3 from the known power coefficient \( S_{w2} \), the parasitic work parameter (default value 0.96), and a specified magnitude for \( S_{w2} \) to allow for inlet swirl if required at the design stage. Aungier (1995) provides an empirical correlation for the parasitic coefficient \( L_e \), the parasitic work parameter (default value 0.96), and a parasitic swirl, \( S_w \), greater than zero, will require a corresponding increase in the magnitude of \( S_{w2} \).

The non-dimensional mass flow, \( \theta_c \), is given by \( m_c U_c a_{1} r_{1}^{3/2} \) and is the fully non-dimensional version of the pseudo non-dimensional parameter \( m_{c'} U_{c'} a_{1} r_{1} / r_{2} \), usually used to present compressor performance. \( \theta_c \) is a measure of the mass flow rate per unit frontal area. The non-dimensional speed \( M_c = U_c / a_{1} \) is usually reduced to \( N / \sqrt{T_{1}} \), \( U_c / a_{1} \) or an equivalent speed when presenting compressor performance. These non-dimensional parameters are based on the inlet stagnation enthalpy or speed of sound. Non-dimensional parameters based on the impeller impeller speed, \( A = \beta_{2B} \), \( \beta_{1B} \), \( b_{1} / r_{1} \), \( r_{1} / r_{2} \) can be obtained by dividing \( S_{w1} \) and \( \theta_{c} \) by \( M_{c} \). As the speed of the impeller must be established it was not considered to be the appropriate parameter to form the basic non-dimensional groups for the procedure presented here.

The design requirement is to obtain the desired geometric parameters, within the bounds of any design constraints, which will lead to the best possible efficiency for the turbocharger unit. The design procedure must arrive at appropriate magnitudes for the non-dimensional mass flow rate, \( \theta_{c} \), the non-dimensional speed \( M_{c} \), and the most appropriate outline geometry. In addition the matching conditions for the turbocharger must also be met. Whitfield (1995).

4 MATCHING REQUIREMENTS
The matching requirements were discussed in detail by Whitfield (1995) and will only be presented briefly. The normal arrangement for a turbocharger is illustrated in Fig. 2.

4.1 Power balance
The turbine must develop sufficient power to drive the compressor, including parasitic disc frictional power for both the turbine and compressor, and overcome the requirements of the bearings. Through the definition of the specific power coefficient, \( S_{w} \), the turbocharger power balance requirement leads to:

\[
S_{w} = \frac{W_{T} M_{c} T_{1} C_{p_{T}}}{C_{c} M_{T} C_{p_{T}}} \tag{3}
\]

where \( S_{w} = U C / h_{n} \) and the subscripts 1 and 2 refer to the impeller inlet and discharge conditions respectively. Specification of a magnitude for \( S_{w2} \) is equivalent to designing with inlet prewhirl. For the usual design with zero inlet swirl \( S_{w1} = \lambda S_{w2} \); however, specification of positive inlet swirl, \( S_{w1} > 0 \), will require a corresponding increase in the magnitude of \( S_{w2} \). The magnitude of \( S_{w1} \) is derived through Eqn.3 from the known power coefficient \( S_{w2} \), the parasitic work parameter (default value 0.96), and a specified magnitude for \( S_{w2} \) to allow for inlet swirl if required at the design stage. Aungier (1995) provides an empirical correlation for the parasitic coefficient \( L_e \), the parasitic work parameter (default value 0.96), and a parasitic work parameter (default value 0.96), and a parasitic swirl, \( S_w \), greater than zero, will require a corresponding increase in the magnitude of \( S_{w2} \).

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4.2 Speed balance
Applying the same rotational speed to the turbine and compressor leads to the speed balance as:

\[
\frac{r_{2T}}{r_{2C}} = \frac{M_{cT}}{M_{cC}} \sqrt{\frac{\gamma R_{c} T_{11c}}{\gamma R_{c} T_{01T}}} \tag{4}
\]

where \( \gamma = \frac{C_{p_{T}}}{C_{p_{c}}} \)

The non-dimensional speed of the compressor is related to the degree of impeller blade backswep and the desired target efficiency. If a desired impeller blade backswep is required, in order to develop an impeller with a broad operating range, the ability to select the non-dimensional speed of the impeller will be limited. As will be seen in Part B if a radial inflow turbine design is the preferred option the ability to select the non-dimensional speed of the turbine will also be restricted. Equation 4 then leads directly to the size of the turbine rotor relative to that of the compressor impeller.

4.3 Mass flow balance
Through consideration of the non-dimensional mass flow it can be shown that:

\[
\frac{\theta_{c}}{r_{2}} = \frac{m_{c}}{m_{r}} \frac{\gamma P_{r1T} P_{r1} P_{r1} P_{r1}}{P_{r1} P_{r1} P_{r1} P_{r1}} \sqrt{\frac{\gamma R_{c} T_{11c}}{\gamma R_{c} T_{01T}}} \tag{5}
\]

The turbine pressure ratio \( P_{r1T} / P_{r1} \) is established from the known power coefficient, \( S_{w} \), and target total to static efficiency for the turbine, see Part B. The pressure ratio \( P_{r1T} / P_{r1} \) represents the stagnation pressure loss in the turbine exhaust pipe, and the pressure ratio \( P_{r1T} / P_{r1} \) represents the stagnation pressure loss in the compressor inlet duct. Equation 7 can then be used to derive the non-dimensional mass flow ratio once the radius ratio, \( r_{2T} / r_{2C} \), has been established through the speed balance. Selection of the non-dimensional mass flow for the compressor has a direct bearing on the design for the turbine rotor. These non-dimensional mass flow parameters are the main parameters to be selected in the optimization of the design of both the compressor and the turbine.

5 IMPELLER DISCHARGE CONDITIONS
The design procedure is based on the establishment of the velocity vectors at inlet to and discharge from the impeller, leading to the design of the component to accept or generate those velocity vectors. The impeller discharge conditions are considered first as they lead to the evaluation of the non-dimensional speed of the impeller \( U_{f} / a_{1} \) which is required for the establishment of the inlet conditions.

Consideration of the impeller discharge velocity vectors, see Fig. 3, leads to the relationship:

\[
\frac{S_{w2}}{\gamma - 1} = \frac{M_{cC} C_{c}}{a_{2s}} \sin \alpha_{2s} \sqrt{1 + S_{wC}} \tag{6}
\]

Consequently it is possible to derive the gas Mach number at impeller discharge for any specified absolute flow angle and a range of non-dimensional impeller speeds. Contours of absolute flow angle can then be plotted as a function of Mach number and non-dimensional impeller speed, Fig. 4.

Consideration of the flow conditions relative to the impeller leads to the relationship:

\[
\frac{S_{w2}}{\gamma - 1} = \frac{U_{f} C_{2s}}{a_{2s}} \tan \alpha_{2s} M_{2s}^{2} \tan \beta_{2s} \tan \beta_{2s} \tag{7}
\]

It is then possible to add contours of discharge relative flow angle to Fig. 4.
5.1 Design selection

This figure effectively provides all the possible discharge velocity vectors for a given power coefficient, \( S_{nc} \), and the designer must select an appropriate design point. This selection will be constrained by non-dimensional speed restraints, desired blade backswep, and guided by published recommendations with respect to an appropriate magnitude for the absolute flow angle. For any non-dimensional speed of the impeller it can be seen that as the absolute flow angle is increased the absolute Mach number decreases. Similarly increasing the absolute magnitude of the relative flow angle, \( \beta_2 \), for any selected absolute flow angle, \( \alpha_2 \), reduces the Mach number. For the particular design illustrated in Fig.4 the non-dimensional impeller speed was restricted to 1.5 through stress considerations and the need to allow for overspeed operation.

Selection of the absolute flow angle can be guided by published data which recommends a magnitude in the range 60° to 70°. This selection is coupled with the need to minimize the Mach number and the appreciation that large magnitudes of \( \alpha_2 \) will lead to a long flow path through the vanless diffuser and, as a consequence, increased friction loss and possibly reduction of the stable flow range.

The graphics procedure allows the designer to scan across the discharge design options and provides a numerical display which continuously up-dates the parameters shown on Fig.4. The parameters of the graph (\( M_2, M_\infty, \alpha_2, \) and \( \beta_2 \)) are provided, together with the relative Mach number, the blade discharge backswep angle \( \beta_{bs} \), the non-dimensional discharge blade height \( b_2/r_2 \), and the specific speed of the impeller. The blade backswep angle \( \beta_{bs} \) is a function of the specified slip factor. The blade height \( b_2/r_2 \) and the specific speed of the impeller are functions of the non-dimensional mass flow rate which, at this stage, must be specified. The actual non-dimensional mass flow rate must be established through consideration of the inlet conditions.

Impeller Blade Angle

The relative flow angle, \( \beta_2 \), is transformed into the impeller blade backswep angle through the application of the slip factor. The definition of the slip factor is given by

\[
\mu = 1 - C_{sl}/U_2
\]

see Fig.3. It can be shown that the relative flow angle and blade angle are related through

\[
\tan \beta_{bs} = \tan \alpha_2 (1 - \mu) + \mu \tan \beta_2
\]

(10)

For any designer specified slip factor the blade backswep angle is displayed and continuously up-dated as the discharge conditions, Fig.4, are surveyed. The slip factor can be readily modified to assess the effect of any uncertainties on the resultant magnitude for the impeller discharge blade angle. Increasing the backswep angle should lead to an improved operating range and efficiency; however, it also leads to an increase in the non-dimensional impeller speed which, if the impeller rotational speed is limited, will lead to an increase in the impeller diameter and inertia.

Impeller Discharge Blade Height \( b_2/r_2 \)

The impeller discharge blade height is derived through the application of the continuity condition at the impeller discharge leading to the non-dimensional mass flow as

\[
\theta_2 = \frac{C_2 \cos \alpha_2 (1 - \gamma - \frac{1}{2} \frac{c_2^2}{\gamma - 1})^{(\gamma - 1)/2}}{\sqrt{1 + S_{nc} \eta_2^{(\gamma - 1)/\gamma}} - 2 \frac{b_2}{r_2}}
\]

(11)

In order to apply this expression it is necessary to specify an efficiency for the impeller; a default value of \( \sqrt{\eta_2} \) is used, but this can be modified by the user. The non-dimensional blade height in the form \( (b_2/r_2)\eta_2 \) can then be derived as the discharge Mach number and absolute flow angle are available for any selected design point on Fig.4. The derivation of the non-dimensional blade height \( b_2/r_2 \) requires the specification of the non-dimensional flow rate; this can be done through the control illustrated at the top of Fig.4, however the final magnitude must be determined through consideration of the discharge conditions.

The effect of any uncertainty associated with the specification of the impeller efficiency is illustrated in Fig.5. With a magnitude for \( \eta_2 \) of 0.1 and an absolute flow angle of 65° the derived non-dimensional blade height decreases by approximately 10% as the assumed efficiency is increased from 0.8 to 0.9. As the design options are searched on Fig.4 the magnitude of \( b_2/r_2 \), for any specified \( \alpha_2 \) and \( \eta_2 \), is displayed continuously up-dated. The magnitude of the impeller passage height \( b_2/r_2 \), which will have an effect on the clearance losses, should be viewed as a flow passage requirement, the geometric height must allow for passage blockage and flow distortion. Aungier (1995) provided a correlation for flow distortion in industrial compressors leading to the fractional aerodynamic blockage.

Impeller Specific Speed

The specific speed is defined as

\[
N_{nc} = \frac{\omega_2 V}{Q^2/34}
\]

(12)

where \( \Delta h_2 \) is the isentropic enthalpy change. With the volume flow rate \( Q \) defined at the inlet stagnation conditions, \( Q = \dot{m}/\rho_1 \), the specific speed can be developed to

\[
N_{nc} = \frac{\pi M_{nc} \omega_2 \theta_2}{\left[(\gamma - 1)/(\gamma - 1)\right]^{3/2}}
\]

(13)

As the pressure ratio \( P_r \) is fixed by the design requirements the specific speed is proportional to the non-dimensional speed \( M_{nc} \), and the square root of the non-dimensional mass flow rate \( \theta_2 \). The non-dimensional speed \( M_{nc} \) is selected through consideration of the design options available on Fig.4. Within any stress and size limitations the selection of the non-dimensional speed will be based primarily on the desired discharge blade backswep angle in order to minimize the discharge Mach number and obtain a broad operating range. To assist with the design selection the specific speed is derived for a specified magnitude of \( \Delta h_2 \) and displayed continuously up-dated as the design conditions of Fig.4 are searched. Rodgers (1993) indicates that specific speeds of the order of 0.7 to 0.8 are required for maximum efficiency, however, where impeller size and inertia are important it may well be necessary to design at increased specific speed. The specific speed and non-dimensional blade height cannot be finally derived until the non-dimensional mass flow rate is established; this must be determined through consideration of the impeller inlet conditions, and, as shown through the matching conditions, has a direct impact on the turbine design.

6 Impeller Inlet Conditions

Through the consideration of the discharge conditions of Fig.4 the non-dimensional speed, \( U_2/a_{in} \), is established. This was based on the discharge swirl parameter \( S_{sw} \) derived through Eqn.3 from the known stage power coefficient \( S_{sw} \) and the inlet swirl conditions represented through the magnitude of \( S_{sw} \). Normally a design is completed for zero prewhirl and, if required, prewhirl is applied through the use of a variable geometry device with no prewhirl applied at the design condition. However, to provide a general design approach inlet swirl, through the specification of a magnitude for \( S_{sw} \), is included here.
6.1 Impeller radius ratio and inlet blade angle

The impeller inducer geometry must be established in terms of the radius ratio, \( r_i/r_o \), and the relative flow angle \( \beta_i \), (which leads to the blade angle through an appropriate incidence angle). The inlet parameters are shown as contours on a graphical presentation of inlet absolute and relative Mach numbers. The inducer tip relative Mach number is a critical parameter on which the design can be assessed as it is desirable to ensure that it is no larger than necessary; if the inducer tip relative Mach number is judged to be too large it may be necessary to re-assess the selected non-dimensional speed of the impeller \( M_{ec} \), or consider the application of prewhirl.

From the geometry of the inlet velocity triangle and the definition of the swirl parameter \( S_i \), it can be shown that the relative velocity is given by

\[
\left( \frac{W_i}{a_{i1}} \right)^2 = \left( \frac{C_{i1}}{a_{i1}} \right)^2 + M_{i}^2 \left( \frac{r_{i1}}{r_2} \right)^2 - 2 \frac{S_i}{1-M_{i}^2}.
\]

The impeller radius ratio contours are a function of the absolute and relative Mach numbers for any non-dimensional speed \( M_{ec} \). For a range of Mach numbers, \( M_{ec} \), the velocity ratio \( C_{i1}/a_{i1} \), can be derived and the relative Mach number established for any desired radius ratio contour.

Without inlet swirl the relative flow angle, from which the blade angle can be established, is given through the inlet velocity triangle as

\[
\cos \beta_i = M_{i1}/M_{irotu}
\]

and leads to relative flow angle contours which are simple straight lines. The resultant contours of both radius ratio and relative flow angle are shown in Fig.6.

With the inclusion of prewhirl it can be shown that

\[
M_{irotu} \cos \beta_i = M_{i1} = \frac{(S_{i2}/(\gamma-1))^2}{M_{irotu}^2 - M_{i1}^2 + 2S_{i1}/(\gamma-1)}
\]

The relative flow angle contours can be derived by specifying a range of Mach numbers and solving Eqn.16 for \( M_{irotu} \), from which the relative Mach number can be derived. The resultant contours are shown in Fig.7 for an inlet swirl parameter of 0.07. These graphical presentations effectively provide the impeller inducer geometry in terms of the inlet absolute and relative Mach numbers. Before these are utilised to provide the impeller design additional parameters can be derived to aid in the design assessment and selection.

6.2 Non-dimensional mass flow contours

As shown in the non-dimensional analysis leading to Eqn.4 the essential parameters to be established are the non-dimensional speed \( M_{ec} \) and non-dimensional flow rate \( \theta_c \). The non-dimensional speed was established through the impeller discharge conditions and it remains to arrive at an appropriate magnitude for \( \theta_c \). It will then be possible to derive the impeller non-dimensional blade height \( \beta_j \), and specific speed, and establish the impeller inlet conditions. With the specification of the design mass flow rate and inlet stagnation conditions the selection of \( \theta_c \) leads directly to the impeller diameter, and through the selected non-dimensional speed \( M_{ec} \) the impeller rotational speed. Through the matching conditions these selections have a direct impact on the turbine design.

Application of the continuity condition at impeller inlet leads to the non-dimensional mass flow rate at impeller inlet as (see Whitfield et al.(1993)).

\[
\theta_c = M_{s1} \cos \alpha_i 2 \left[ \frac{r_1}{r_2} \left( 1 + \frac{v^2}{1 - \frac{\gamma - 1}{2} M_{s1}^2} \right)^{(\gamma-1)/2} \right]^{(1/\gamma - 1)}
\]

where \( v \) is the inducer hub to tip radius ratio \( r_i/r_o \), and must be specified. This is usually based on structural and installation requirements, the hub size requiring sufficient circumferentence to carry the desired number of inducer blades without excessive blockage. Equation 17 assumes that the inlet prewhirl, if any, is of the free vortex type so that the axial component of velocity does not vary with radius.

The shroud radius ratio \( r_o/r_i \) is related to that at the mean through

\[
(r_i/r_o)^2 = 2(r_i/r_o)^2((1 + v^2)
\]

For any desired \( \theta_c \) contour Eqn.17 can be solved for radius ratio by specifying a range of Mach numbers; the relative Mach number can then be found through Eqn.14.

The contours of \( \theta_c \) together with \( r_i/r_o \) and \( \beta_i \) are shown in Fig.6 for a non-dimensional speed \( M_{ec} = 1.5 \) and \( v = 0.34 \). The designer must select an appropriate design point on this plot bearing in mind the need to maximize the efficiency, whilst meeting other design constraints such as impeller speed, size and inertia requirements, and the need for a broad operating range. The graphical presentation enables the designer to scan across the options of Fig.6 and continuously updates the parameters of the graph together with those appropriate for design assessment, see the next Section. From Fig.6 it can be seen that for any non-dimensional flow contour, \( \theta_c \), there exists a minimum relative Mach number condition with an associated radius ratio and relative flow angle. In order to obtain a broad operating range between the design point and choke it is necessary to maximize the margin between the design point and relative Mach number and unity. Minimizing the relative Mach number by selecting a small magnitude for the non-dimensional flow rate will lead to a relatively large impeller diameter, this will not only lead to increased rotor inertia but also to a long narrow flow channel and reduced efficiency. The specific speed correlations of Rodgers(1993) provide a good guide to the magnitude of non-dimensional mass flow rate. Assuming that \( S_{i2} = S_{ec} \) and combining Eqsns.13 and 9 it can be shown that the specific speed is given by

\[
N_{ac} = \frac{n(\tan \alpha_1 - \tan \beta_{i2})/(\mu \tan \alpha_2)}{\eta_c ((P^{i+1} - 1)/(\gamma - 1))^{1/2} \theta_c}
\]

This expression is shown graphically in Fig.8 where the effect of discharge blade backwept is illustrated for a pressure ratio of 3.6, and the effect of pressure ratio shown by an increase to 8.0. Specific speeds in the range 0.7 to 0.8 lead to \( \theta_c \) magnitudes in the range 0.08 to 0.1. Specific speeds of 1.0 lead to a \( \theta_c \) of the order of 0.15. Once the design parameters at impeller discharge have been selected the specific speed is a function of \( \theta_c \) only and can be readily tracked and presented as the inlet design options are considered.

The selection of the non-dimensional mass flow, \( \theta_c \), together with a specificaition of the design mass flow rate and inlet stagnation conditions leads directly to the impeller diameter and, when combined with the non-dimensional speed \( M_{ec} \), the rotational speed. Impeller diameter and rotational speed is presented and continuously up-dated as the designer scans across the options presented on Fig.6.

6.3 Design assessment

Whilst surveying the design options of Fig.6 or 7 a number of additional parameters can be made available to the designer so that informed decisions can be made. As shown above the specific speed is available with the selection of \( \theta_c \). Through the matching conditions the specific speed of the turbine is also available, see Part B.

**Prewhirl angle**

If the design has been conducted with the inclusion of prewhirl a magnitude for \( S_i \) has to be specified; the consequences of this in terms of the swirl angle required must be assessed. From the definition of \( S_{i1} \) it can be shown that
As the design options of Fig. 7 are surveyed the prewhirl angle can be displayed and continuously updated. As designs with prewhirl are not common the option to provide the swirl angles as contour plots has not been provided. The magnitude of the swirl angle is related to the defined swirl parameter \(S_p\); it can be seen from Fig. 9 that if the desired design point swirl angle is not to exceed 20° then the magnitude of \(S_p\) is of the order of 0.04.

### Relative velocity ratio

The performance of an impeller is closely related to the rate of diffusion of the relative velocity through the impeller passage. The relative Mach number ratio is often adopted as a diffusion rating technique, see Japikse (1986). As the impeller inlet conditions are surveyed the ratio of the Mach numbers, \(M_{inc}/M_{dis}\), can be derived and displayed and can be used to assess the likely performance of the proposed design. The relative velocity ratio is also used as a parameter to assess the rate of diffusion through the impeller passage, Dean (1972). The relative velocity ratio, \(W_p/W_t\), can be derived through the combination of

\[
W_p = W_t \cos \alpha_2 = \frac{S_{22}}{\gamma - 1} \frac{M_a \tan \alpha_2 \cos \beta_2}{a_{11}}
\]

and

\[
W_p = \frac{M_{inc}}{M_{dis}} \frac{a_{11}}{\sqrt{1 + \frac{1}{2} \frac{M_{dis}^2}{M_{inc}^2}}}
\]

The relative velocity ratio \(W_p/W_t\), when used for axial flow compressors is referred to as the de Haller number, and Wilson (1984) suggests that this should not be less than 0.7. For a centrifugal compressor where the flow must pass through a bend from the axial to radial direction Wilson suggests that if separation is to be avoided the relative velocity ratio limit should be increased from 0.7. However the design constraints of many centrifugal compressors are such that a high peripheral speed is necessary and the relative velocity ratio is such that separation occurs within the impeller passage. The magnitude of the relative velocity ratio and relative Mach number ratio is continuously displayed to aid the design selection as the inlet conditions are surveyed on Fig. 6.

### Diffusion Factor

As an alternative to the relative velocity ratio a diffusion factor can be derived and continuously displayed as the options available for the inlet conditions are surveyed provided the number of blades is specified, see Whitfield et al. (1993). Rodgers concluded that optimum impeller designs would have stall point diffusion factors below 0.75.

### 7 SUMMARY OF THE DESIGN PROCEDURE

The non-dimensional design is based on a specified compressor pressure ratio and target efficiency which are combined to provide a non-dimensional power coefficient, from which the compressor and turbine design is developed. The only additional empirical data required are the impeller slip factor and efficiency which can be readily modified to assess any uncertainties involved. The specified slip factor has a direct bearing on the derived impeller discharge blade angle, and the impeller efficiency on the derived non-dimensional discharge blade height of the impeller.

The procedure essentially provides the designer with all the options available at impeller inlet and discharge in a graphical format which can be searched and assessed. This assessment must be based on all the design requirements and constraints, the optimum requirements for the impeller, the requirements for the downstream diffuser and collecting system, and the requirements for the upstream ducting and any guide vanes if adopted. In addition the compressor design must ensure that the constraints imposed by the matching requirements of the drive turbine lead eventually to the design of an optimum turbine and optimum turbocharger or gas generator unit.

Selection of the most appropriate design is left in the hands of the designer as the many requirements and constraints differ from application to application, many of which cannot be quantified solely in terms of a one-dimensional aerodynamic design. To aid in the design selection parameters such as specific speed, critical Mach numbers and diffusion ratios are displayed and continuously updated as the design options are searched.

For consideration of the turbine matching the engine air fuel ratio, turbine inlet temperature relative to that at compressor inlet, and a target efficiency for the turbine must be specified; these are introduced in Part B.

### 8 REFERENCES

- Abidat M et al. 1992
  Design of a highly loaded mixed flow turbine
  Proc Instn mech Engrs A v.206 pp.95-107
- Aungier R H 1995
  Centrifugal compressor stage preliminary aerodynamic design and component sizing
  ASME Paper No. 95-GT-78
- Chou C and Gibbs C A 1989
  The design and testing of a Mixed flow turbine for turbochargers
  SAE Paper No. 890644
- Dean 1972
  The fluid dynamic design of advanced centrifugal compressors
  Creare Inc. TN-153 Sept. 1972
- Japikse D and Osborne C 1996
  ASME Paper No. 86-GT-222
- Rodgers C 1978
  A diffusion factor correlation for centrifugal impeller stalling
  Trans ASME J Engng Fwr, 100 p.592
- Rodgers C 1991
  The efficiencies of single stage centrifugal compressors for aircraft applications
  ASME paper No. 91-GT-77
- Rodgers C 1993
  Technology requirements for small gas turbines
- Wilson D G 1984
  The design of high-efficiency turbomachinery and gas turbines
  The MIT Press
**Fig. 1** Radial Flow Compressor

**Fig. 2** Turbocharger System

**Fig. 3** Discharge Velocity Vectors

**Fig. 4** Discharge Contours

**Fig. 5** Effect of Efficiency on $b/r_2$
Fig. 6 Inlet Contours with Zero Swirl

Fig. 7 Inlet Contours with Preswirl

Fig. 8 Non-dimensional Mass Flow

Fig. 9 Effect of Swirl Parameter on Swirl Angle