INVESTIGATION OF THROUGH-FLOW HYPOTHESIS IN A TURBINE CASCADE USING A 3D NAVIER-STOKES COMPUTATION

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Abstract

The results of a computation, performed with a three-dimensional Navier-Stokes computation at ONERA, have been averaged in the blade-to-blade direction; the spatial fluctuations around the averaged flow variables have also been determined. It has then been possible to estimate all terms in the average components of the momentum equations. The comparison of the two-dimensional balances of these three equations shows that the shear stress play a minor role in the momentum balance, except on the dissipation of the passage vortex kinetic energy downstream of the blade trailing edges. The kinetic energy of the spanwise component of the velocity spatial fluctuations has a very strong influence on the radial pressure gradient; it introduces a convection effect. This is a key effect for all these balances.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>b</td>
<td>blade blockage in the z direction</td>
</tr>
<tr>
<td>g</td>
<td>distance between two consecutive blades, in the z direction, outside the blade passage</td>
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<tr>
<td>K</td>
<td>kinetic energy of the spatial fluctuation</td>
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<tr>
<td>P</td>
<td>static pressure</td>
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<tr>
<td>Re</td>
<td>Reynolds number</td>
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<tr>
<td>T</td>
<td>Temperature</td>
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<tr>
<td>u</td>
<td>velocity</td>
</tr>
<tr>
<td>x, y, z</td>
<td>axial, spanwise, and blade-to-blade directions</td>
</tr>
<tr>
<td>p</td>
<td>static density</td>
</tr>
<tr>
<td>τ</td>
<td>shear stress</td>
</tr>
<tr>
<td>n</td>
<td>unit vector in z direction</td>
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Superscripts

- Mass average in the z direction (see equation 2)
- Fluctuation in the z direction around the mass average (see equation 3)
- Area average in the z direction
- Fluctuation in the z direction around the area average

Introduction

The computation of the flow in blade passages of compressors and turbines may be performed with efficient three-dimensional Navier-Stokes codes, which give access to very fine local flow structures in only a few hours on a vector computer (Escande, Cambier, 1991; Dawes, 1987). The practical treatment of the large amount of flow data is however a severe limitation for the use of such tools, particularly if some global information is looked for during the design process for example. Simpler tools are then needed, such as the through-flow computation. This model solves the flow equation in a two-dimensional form, and can also handle a whole multistage machine in a single pass. Consistent through-flow models with three-dimensional Navier-Stokes solution are also needed. They may be based on various averages in the blade-to-blade direction, the most useful and coherent of which seems to be the mass average procedure (Hirsch, Dring, 1987). A whole set of mass average Navier-Stokes equations have been given by Hirsch (1990).

The main limitation of through-flow computation is its apparent inability to reproduce three-dimensional effects, of which there are three types: convection, diffusion due to shear stress and heat flux, and blade pressure and friction forces. Some phenomena may however be simulated, provided special models are employed. For instance, the passage vortex is linked to the action of the blade pressure force on the end-wall shear layer. If the pressure field is not too much distorted by the vortex, then a boundary layer hypothesis may be used for the end-wall layer computation. Similarly, in an inviscid flow, the information about the pressure force may be replaced by the flow deviation. The secondary flow deviation, which is an aspect of the three-dimensional convection effect, may then be easily computed with the help of a vorticity transport equation. In this equation, the explicit influence of the pressure is naturally weak, and appears implicitly through the inviscid flow deviation. Leboeuf and Brochet (1985) have obtained some good results inside blade compressor passages. However, because this type of method does not take into account three-dimensional convection or diffusion, it forces the secondary vorticity and also the losses, to be confined to the end-walls. This behaviour is a severe limitation for the computation of multistage machines.
Supplementary models have been derived which may account for a so-called "radial mixing" process. Two basic approaches have been followed. Adkins and Smith, (1982) assume a model, based on "the convection of fluid properties by the secondary flow field" with various contributions of the passage vortex, the leakage clearance effects, the centrifugal effect on the blade boundary layer and wake. On the opposite, Gallimore and Cumpsty (1986) assume a diffusion model for the radial mixing process. In practice, both models introduce diffusion terms in the through-flow equation. A mixing diffusion coefficient has to be adjusted to fit the experimental data. Dring (1992) has shown that, even for very high values of the mixing coefficient, neither of the previous radial mixing models were able to correct the discrepancy between the measured blade pressure force and the measured momentum changes across the blade row for his two-stage axial flow compressor. More important, according to this author, the impact of these models "is small, and it is not in general driving it toward the measured results." In his conclusion, Dring suggests the possible strong influence of corner stall on his test case.

Leyleik and Wisler (1991) have performed a three-dimensional Navier-Stokes computation in axial-flow compressors. In their answers to the written discussion of the paper, they remark that "there is general agreement ... that both secondary flow (three-dimensional convection effects) and turbulent diffusion can play important roles in the mixing process."

The objective of this paper is to estimate the relative importance of the various terms which occur after an azimuthal average is applied on the Navier-Stokes equations. For that purpose, we have performed a numerical experiment on a three-dimensional Navier-Stokes code. The results of a computation, performed with a three-dimensional Navier-Stokes computation at ONERA, have been used. For the test case, we use a turbine profile with a high deviation capability; this enables a strong three-dimensional distortion of the flow. However, a cascade with straight blades was chosen, to focus on the main flow features related to viscous and turbulent effects. The results of the Navier-Stokes computation have been averaged in the blade-to-blade direction. The various terms of the through-flow momentum equations have been computed. Our purpose is to show how three-dimensional effects will disturb the averaged blade-to-blade quantities, with particular emphasis on convection, pressure and shear driven phenomena.

The test case and the Navier-Stokes computation

We have used the test turbine proposed by Denton, Hodson and Dominy (1990). This is a cascade of turbine blades with a profile typical of the root section of a low pressure aircraft gas turbine. It has been tested in the variable density cascade tunnel at the Whittle Laboratory in Cambridge (UK). The experimental conditions are: overall deviation of the flow is 92.4°, inlet Mach number is 0.5, and isentropic exit Mach number is 0.71, inlet Reynolds number is 2.3 10^5.

The computation has been performed by ONERA with the code CANARI. This method has been developed by different authors at ONERA. (Cambier, Couaillier and Veuillot, 1988). The numerical method is characterised by an explicit centred finite difference scheme of Lax-Wendroff type, with two steps, associated with a multigrid accelerator. Artificial viscosity of second and fourth order is added to the equation, to ensure numerical stability. Boundary conditions as well as connections between sub-domains are treated with compatibility relationships.

The code is used here with a fine mesh, which allow an accurate description of the walls' geometry. The splitting of the domain in an O-type mesh around the blade and two H-type sub-domains upstream and downstream allows an accurate description of the rounded trailing and leading edges. The sub-domains have respectively 26875, 262605 and 42875 meshes, from upstream to downstream, on the half blade span. The location of the mesh points in the laminar sub-layer enables the capture of very small vortex structures. The turbulence model is the mixing length model of Michel (1969), corrected for three-dimensional geometries by the model from Buleev (1962). Although it is a very simple turbulence model, it produces a good qualitative picture of the flow behaviour. All the flow quantities, and the terms in the equations presented in this paper, have been non-dimensionalised by references quantities: temperature Tref = 293 K, density p ref = 1.293 kg/s, velocity Vref = 341.1 m/s, length Lref = 0.05253 m. The fluid is air.

This code has been validated on various fundamental and turbomachine cases, including turbines with similar meshes (Cambier and Escande, 1989, Escande and Cambier, 1991). The predictions appear to be of good quality. As it is not our purpose to check the accuracy of the predictions against experimental results, we shall assume that these numerical data produce a good qualitative simulation of the local flow structures.

The blade-to-blade average

The results of the computation have been mass averaged in the blade to blade direction z, which stands for the circumferential direction 8 in a cylindrical frame of reference. As the turbine blades are arranged in a straight fixed cascade, we use a Cartesian frame where x is the axial direction and y is a direction parallel to the blade span.

The mass averaging process is described by the following formulas. If A(x,y,z) is a flow variable, A̅ is the related mass weighted spatial average and A' is the spatial fluctuation of A in the z direction

\[ A̅(x,y,z) = \overline{A}(x,y) + A'(x,y,z) \] (1)

\[ \rho A̅ = \rho \overline{A} = \frac{1}{b} \int_{0}^{b} \rho A(2/g) \] (2)

where g is the blade pitch and b is the fraction of space free of blades. The variable A stands for any flow variables, except for the density \( \rho \), the pressure P and the shear stess for which equation (2) is replaced by a simple area average. In that case, the following notation are used:

\[ \rho (x,y,z) = \rho (x,y) + \rho''(x,y,z) \]

\[ P(x,y,z) = P(x,y) + P''(x,y,z) \]

Using this averaging process on the gradient operator, we get:

\[ \nabla A = \frac{1}{b} \nabla (bA) + \frac{1}{b} \Delta [A]_{ps} \] (3)

where \( \nabla \) is the gradient operator in the (x, y) plane, and \( \Delta \) is the difference operator. The last term of equation (3) stands for a blade effect; it is the origin of the blade pressure and shear force in the momentum equations.

The average of the non-linear terms in the transport equation will introduce some new unknowns, from the point of view of the averaged quantities.
The last term in equation (4) is derived according to equation (2); it accounts for the non-uniformity of the flow in the blade-to-blade direction, which is influenced by the three-dimensional nature of the flow.

This averaging process is then applied to the three components of the momentum equation. It is described in the following paragraphs.

The kinetic energy of the spatial fluctuation

The local value of the spatial fluctuation kinetic energy \( K \) is defined as:

\[
K = K_x + K_y + K_z = \frac{1}{2} \left( u_x^2 + u_y^2 + u_z^2 \right)
\]

and the averaged value:

\[
\bar{K} = \bar{K}_x + \bar{K}_y + \bar{K}_z = \frac{1}{2} \left( \bar{u}_x^2 + \bar{u}_y^2 + \bar{u}_z^2 \right)
\]

Figure 1: Averaged kinetic energy \( \bar{K} \) of the spatial fluctuation in the \( (x, y) \) surface (LE and TE stand for leading edge and trailing edge respectively).

Figure 1 displays the evolution of the averaged kinetic energy \( \bar{K} \) in the \( (x, y) \) plane. Figure 4 (a) to 4 (e) give five representations of the local quantity \( K \) in \( (y, z) \) planes. The locations of these planes are given by the symbols A to E on figure 1, with the following axial locations \( x/c = 0.1, 0.15, 0.98, 1.0, 1.1 \).

We consider first the structure of the spatial fluctuation kinetic energy \( K \). At mid-span, the flow is two-dimensional \( (x, z) \), because it is a symmetrical plane. Figure 2 gives the axial evolution of the averaged kinetic energy \( \bar{K} \) at mid-span. Upstream the leading edge, \( \bar{K} \) strongly increases until a maximum occurs at the leading edge, then it decreases. This phenomenon results from the blade surrounding by the flow. This is a potential effect. Figure 1, we can notice that this effect is reinforced near the end-wall by the boundary layer separation that occurs in front of the leading edge. In the blade passage, \( \bar{K} \) is maximum just after mid-chord and then strongly decreases. This has two origins: the boundary layer development on the blade walls, and the blade suction side acceleration that is a consequence of the curvature. The blade to blade evolution at mid-span of the local spatial fluctuation kinetic energy is given figure 3 for the stations A to C defined above. Clearly the suction side acceleration effect is dominant in stations A and B, while near the trailing edge (station C).
side corner, a strong dissipation and a strongly reduced velocity magnitude exists. Furthermore near the suction side, but displaced away from the end-wall along the blade, a maximum exists; this area is influenced by a strong passage vortex that interacts with the blade wall, with a corresponding strong increase in velocity magnitude near the wall. At the trailing edge, the averaged fluctuation kinetic energy suddenly increases all over the blade span (figures 1 and 2). The passage vortex effect and the suction side corner one are superposed on wake effect (figure 4 (d)). As seen on figure 4 (d and e), and on figure 1, the secondary kinetic energy K strongly decreases after the trailing edge, particularly in the wake, while the passage vortex is still detected 50% of chord downstream.

The averaged $y$ component equation: the "radial equilibrium" equation

The averaged $y$ component of the momentum equation is written as:

\[
\frac{\partial}{\partial x} \left( -bp \frac{u_x u_y}{y} \right) + \frac{\partial}{\partial y} \left( -bp \frac{u_y^2}{y} \right) = \frac{\partial}{\partial y} \left( \frac{-bp u_x u_y}{y} \right) - \frac{\partial}{\partial x} \left( \frac{-bp u_y^2}{y} \right)
\]

\[
+ \frac{1}{Re} \left( \frac{\partial}{\partial x} \left( b \gamma_{xy} \right) + \frac{\partial}{\partial y} \left( b \gamma_{yy} \right) \right)
\]

\[
\frac{1}{Re} \left\{ \Delta [ \gamma_{xy} \frac{\partial z}{\partial x} ]_{ps} + \Delta [ \gamma_{yy} \frac{\partial z}{\partial y} ]_{ps} - \Delta [ \gamma_{xy} ]_{ps} \right\}
\]

(6)

The blades are not twisted, and so $\partial z/\partial y = 0$ on the blade surfaces. As the no-slip condition is used in the three-dimensional computation, the average of the convective transport terms does not introduce any contribution on the blade surfaces. The different terms of equation 6 are written as follow:

- Convection terms
- Pressure terms + Fluctuation terms + Shear stress terms

Figure 5 gives the $y$ momentum balance at station B defined in figure 1. Figures 5 (a) to (c) gives the contributions of the individual terms to the balance of equation (6). We have not presented the shear stress terms, because their contributions are significant only near the end-wall as we can see figure 6. We have found from the numerical computations that the following terms are negligible compared to the others:

\[
\frac{\partial}{\partial y} \left( -bp \frac{u_y^2}{y} \right) = 0, \quad \frac{\partial}{\partial x} \left( -bp \frac{u_x u_y}{y} \right) = 0
\]

\[
\left\{ \Delta [ \gamma_{xy} \frac{\partial z}{\partial x} ]_{ps} + \Delta [ \gamma_{yy} \frac{\partial z}{\partial y} ]_{ps} - \Delta [ \gamma_{xy} ]_{ps} \right\} = 0
\]

(7)

From the figures 5, we may conclude that the convective terms (a) are negligible compared to the pressure (b) and fluctuation terms (c). The pressure term (b) is then only influenced by the fluctuation terms (c); this is a consequence of the passage vortex. This can be understood by considering the
Figure 6: Y momentum balance at station B along y direction.

Figure 7: Static pressure contours at station E.

Figure 8: Local shear stress component $\tau_{xy}$ at station C.

Static pressure contour station E (figure 7). We observe a strong distortion of static pressure.

Figure 8 gives the local shear stress component $\tau_{xy}$ in station C. As mentioned above, the averaged shear stress is significant only near the endwall. Nevertheless, figure 8 shows that even the shear term is negligible in equation (6), the local shear stress effect may be important.

From this analysis, we conclude that the influence of the three-dimensional flow structures on the y momentum balance (the "radial equilibrium" equation) is very strong, as can be seen on the pressure gradient variations (figure 5(b)). The pressure varies along y mainly under the influences of the gradient along y of the fluctuation kinetic energy $K_Y$. A second important conclusion from these computations is the negligible role of the shear stress terms in the y momentum balance except very near the endwall.

These remarks may suggest some guidelines for through-flow modelling. First, the importance and the convective nature of the spatial fluctuation effect, described in figure 5(c), should question the exclusive use of diffusive models for the so-called "radial mixing" effects. Second, the present results show clearly that shear stress terms are negligible almost everywhere in the y averaged momentum balance. Finally, trailing edge phenomena are dominated by 3D convection terms and their dissipation. We conclude that shear stress effects mainly occur through the dissipation of the spatial fluctuations.

The averaged z component equation: the "blade-to-blade" equation

The z component of the average momentum equation is given below:

$$
\frac{\partial}{\partial x} (\rho \frac{\partial u}{\partial x} u_z) + \frac{\partial}{\partial y} (\rho \frac{\partial u}{\partial y} u_z) = -\Delta \left[ \rho \frac{\partial}{\partial x} (b \tau_{xz}) + \frac{\partial}{\partial y} (b \tau_{yz}) \right] + \frac{1}{Re} \left\{ \frac{\partial}{\partial x} \left( \rho \frac{\partial u}{\partial x} \right) \right\} - \frac{1}{Re} \left\{ \frac{\partial}{\partial y} \left( \rho \frac{\partial u}{\partial y} \right) \right\} - \Delta \left[ \frac{\partial^2 \rho}{\partial x^2} \right] \right]
$$

(8)

We have found from our numerical computations that the gradient along y of the momentum is negligible compared to the x one. As in the previous paragraph, the blade shear stress terms are negligible too. Figure 10 presents the z momentum balance station B and figures 9(a) to (c) the evolution of the individual terms in the z momentum equation in (x,y) plane. The blade pressure force (figure 9(b)) is maximum at mid-chord. Figure 10 shows that the blade pressure term has a strong counterpart in the gradient along y of the momentum. This momentum gradient is also influenced by the shear stress terms near the end-wall and by the spatial fluctuation terms everywhere (about 10%). The evolution of the fluctuation terms (figure 9(c)) has three origins: a two-dimensional effect described above (figure 2), an important contribution of the suction side corner effect, and a slight influence of the passage vortex.

The local component of the shear stress tensor are not presented in detail here, for a lack of space; however their overall behaviours are very well represented by the $\tau_{xy}$ value in figure 8.

We conclude that the spatial fluctuation terms have a small but not negligible influence on the balance of the z momentum equation. However, the three-dimensional effects have a stronger indirect control on this equation. They modify the pressure gradient in the y equation (figure 5(b)). This has some consequence on the blade pressure force, particularly near the end-wall (figure 9(b)); in particular, the location of the passage vortex core very near the blade suction side (figure 7), is responsible for the high variation of the blade force. The shear stresses have a negligible direct effect on the z momentum balance except near the endwall.
Figure 9: Contributions of the most important terms in the $z$ component of the momentum equation (8): (a) convection term, (b) blade pressure term, (c) spatial fluctuation term.

Figure 10: $z$ momentum balance at station B along $y$ direction.

Figure 12: Contributions of the most important terms in the $x$ component of the momentum equation (9): (a) convection term, (b) axial pressure gradient and pressure blade force, (c) fluctuation term.

The averaged axial $x$ component equation

The $x$ component of the average momentum equation is given below:

$$
\frac{\partial}{\partial x} \left( - \beta \left( \frac{\partial u_x}{\partial x} \right) - \frac{\partial}{\partial y} \beta \left( \frac{\partial u_x}{\partial y} \right) \right) =
\frac{\partial}{\partial x} \left( \beta P \right) + \Delta \left[ \frac{\partial}{\partial x} \left( \frac{\partial u_x}{\partial x} \right) \right] \beta \left( \frac{\partial u_x}{\partial x} \right) - \frac{\partial}{\partial y} \beta \left( \frac{\partial u_x}{\partial y} \right)
+ \frac{1}{Re} \left[ \frac{\partial}{\partial x} \left( \frac{\partial \tau_{xx}}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial \tau_{xy}}{\partial x} \right) \right] + \frac{1}{Re} \left[ \Delta \left( \frac{\partial \tau_{xx}}{\partial x} \right) + \Delta \left( \frac{\partial \tau_{xy}}{\partial y} \right) \right] \Delta \left( \tau_{xx} \right) \right)
(9)

Figures 12 (a) to (c) give the contributions of the most important terms in the $x$ components of the momentum equation (9): (a) convection term, (b) axial pressure gradient and pressure blade force, (c) the fluctuation terms.

As previously, the blade shear stress terms are negligible, and the axial averaged shear stress gradient is significant only near the end-wall. On the whole, there exists an approximate balance between the pressure term (b) and the convection term (a). We can notice that the axial pressure
gradient and the pressure blade forces have the same order of magnitude in the pressure term (b), although their contributions are not shown here. Moreover the three-dimensional influence is more important on the pressure blade force than on the axial pressure gradient. Figure 11 presents the axial evolution of the momentum balance at mid-span. It shows that, in the blade passage, the fluctuation terms are of the order of 10 to 20% of the other terms, with strong peaks at the leading and trailing edges. Their origins are linked to the suction side acceleration describe in the paragraph on the spatial fluctuation kinetic energy (figure 2).

Conclusion: Some comments about axi-symmetrical flow computation.

From the previous momentum balance analysis, we may suggest some guidelines for axi-symmetrical flow modelling.

First we show that the spatial fluctuation influences are important in the $x$, $y$, and $z$ components of the momentum equation. We have shown that the convection term is more important than on the axial pressure gradient. Figure 11 presents the axial evolution of the momentum balance at mid-span. It shows that, in the blade passage, the fluctuation terms are of the order of 10 to 20% of the other terms, with strong peaks at the leading and trailing edges. Their origins are linked to the suction side acceleration described in the paragraph on the spatial fluctuation kinetic energy (figure 2).

Acknowledgements

The authors like to thank Mr G. MEAUZE and G. BILLONNET from ONERA, Chatillon, for having performed this three-dimensional Navier-Stokes computation. Their help and interest throughout this work has been strongly appreciated.

We would also like to thank EDF, SEP, SNECMA and TURBOMECA, which, as partners of a Firtech organisation, have provided part of the financial support for the grant of G. Perrin.

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