CONDENSATION IN JET ENGINE INTAKE DUCTS DURING STATIONARY OPERATION

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ABSTRACT

The paper describes an analysis of the condensation of moist air in very long intake ducts of jet engines during stationary operation. Problems arising from such condensation include fan overspeed and increased stagnation pressure loss in the intake duct.

The analysis demonstrates that, for moderate values of relative humidity, homogeneous condensation will occur in an outer annulus adjacent to the intake cowling if the local flow Mach number attains values of about 1.0. In the central region of the intake duct, where design Mach numbers of 0.8 may be attained, homogeneous condensation is unlikely to occur except, possibly, when the relative humidity is close to 100% and the ambient temperature very high. However, if the intake duct is very long, significant heterogeneous condensation on foreign particles present in the atmosphere is possible. The concentration of foreign nuclei required for this type of condensation is comparable to the likely levels of contamination at many industrial test sites.

The effects of condensation on engine test results are twofold. Firstly, condensation is a thermodynamically irreversible process and results in an increase of entropy and hence loss of total pressure in the intake duct. Uncorrected measurements using Pitot probes may not record this loss correctly. Secondly, the mass and energy transfer between the phases during the condensation process has a tendency to accelerate the flow approaching the engine, an effect which may be counteracted by a reduction in mass flowrate in order to maintain the static pressure constant. These conclusions are in agreement with experimental results obtained on-site during the testing of a jet engine fitted with a very long intake duct.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>a</td>
<td>Speed of sound</td>
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<tr>
<td>$c_p$</td>
<td>Isobaric specific heat capacity</td>
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<td>g</td>
<td>Mass fraction of air</td>
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<tr>
<td>h</td>
<td>Specific enthalpy</td>
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<td>$h_{fg}$</td>
<td>Specific enthalpy of evaporation</td>
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<td>J</td>
<td>Homogeneous nucleation rate per unit volume</td>
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<td>k</td>
<td>Boltzmann’s constant</td>
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<td>M</td>
<td>Mach number</td>
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<tr>
<td>m</td>
<td>Mass of a water molecule</td>
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<td>p</td>
<td>Pressure</td>
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<td>R</td>
<td>Specific gas constant</td>
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<td>$r_*$</td>
<td>Kelvin-Helmholtz critical radius</td>
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<td>S</td>
<td>Supersaturation ratio</td>
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<td>s</td>
<td>Specific entropy</td>
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<td>T</td>
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<tr>
<td>$\Delta T$</td>
<td>Vapour subcooling</td>
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<td>t</td>
<td>Time</td>
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<tr>
<td>V</td>
<td>Velocity</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Wetness fraction (mass fraction of liquid H$_2$O)</td>
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<tr>
<td>$\gamma$</td>
<td>Ratio of specific heat capacities</td>
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<tr>
<td>$\phi$</td>
<td>Relative humidity</td>
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<tr>
<td>$\rho$</td>
<td>Density</td>
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<tr>
<td>$\sigma$</td>
<td>Surface tension</td>
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Subscripts:

e = equilibrium
f = frozen

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INTRODUCTION

It is well known that the condensation of humid air can cause serious limitations to the working range of transonic and supersonic wind tunnels (Wegener, 1966). Similar phenomena can also restrict the stationary operation of jet engines (Blake, 1975, Zerkle et al., 1982). The effects are particularly pronounced in configurations involving very long intake ducts such as those associated with tail-mounting, above the fuselage, on large three-engined civil airliners and purpose built test cells for stationary engine testing. This paper presents a general theoretical analysis of the effects of homogeneous and heterogeneous nucleation of water vapour in the intake system. The conclusions are consistent with on-site measurements from a stationary jet-engine test conducted by Rolls-Royce. The analysis also provides guidelines for assessing the effects of unwanted condensation under any particular test condition.

It is important to appreciate that the results apply solely to the case of stationary operation on the ground. The same conclusions do not apply under normal flight conditions, even at low altitude where the humidity may be high, because of the change in the flow stagnation conditions relative to the aircraft.

BACKGROUND

Figure 1 shows a typical long inlet duct feeding a high bypass ratio jet-engine. The duct is straight with diameter 2.4 m and length (from the inlet to the engine face-plane) 8.5 m. The inlet cowling is specially designed to provide a smooth intake for the flow and the velocity in the long straight section is (with the exception of the boundary layer) near-uniform across the duct for most of its length. On conventional designs, the Mach number in this section is usually about 0.6 but increased engine thrust can be obtained without increase of intake diameter if the Mach number is increased to about 0.8. It is at these high intake Mach numbers that problems associated with condensation are most marked.

Figure 1 also shows the computed Mach number contours for stationary operation with dry air and a Mach number of about 0.8 in the duct. Details of the flowfield vary with the geometry of the inlet but the important characteristics are common to most designs. Along the centre-line of the duct the Mach number rises smoothly and monotonically to 0.8 and remains approximately constant to the engine face-plane. Close to the lip of the cowling (but outside the boundary layer) the Mach number rises to about 1.1 (due to the high curvature of the streamlines) and then falls to the constant value of 0.8. Figure 2 shows typical pressure distributions along centre-line and cowling wall streamlines, plotted as functions of distance along the streamlines.

It is important to appreciate that, because phase change proceeds at a certain rate, the effects of condensation are determined by absolute time scales and are therefore related, not only to the local Mach number, but also to its rate of change. Figure 2 shows that the rate of expansion near the cowling wall is much higher than along the centre-line of the duct. It might therefore be expected that the characteristics of condensation close to the cowling are rather different from those near the centre-line and this indeed proves to be the case. It should also be noted that the residence time of a fluid particle in the long straight section of the intake (8 m at a Mach number of 0.8) is about 30 ms.
A full-scale configuration similar to the one shown in Fig. 1 was tested by Rolls-Royce at their experimental test facility at Hucknall in the U.K. Hucknall is situated near Nottingham, a large industrial city with a rather damp climate. The following results were obtained at design conditions:

1. A fan overspeed of a few percent was recorded.
2. Pitot probes installed in the intake duct near the engine face-plane and outside the wall boundary layer recorded stagnation pressure losses along the length of the duct of the order of 2 - 3%. (Outside the boundary layer, the expansion to the engine face-plane was expected to be near-isentropic).
3. Condensed water was discovered in these same pitot tubes.

Similar experiments performed at a test site in the United States subject to a very dry climate showed none of these anomalies.

It was clear from the presence of water in the probes that condensation was occurring in the intake during the tests with humid air. Close to the intake cowling, the flow was transonic and it was suspected that homogeneous nucleation of water droplets might be occurring in this region. In the central core of the flow, where the Mach number was lower, the condensation was more probably heterogeneous in nature.

Subsequent analysis confirmed these hypotheses and also demonstrated that condensation can give rise to effects which may seem disproportionate considering the very low mass fractions of water vapour actually present. To underline this point, Fig. 3 shows the mass fraction of water vapour as a function of the relative humidity and temperature. Apart from very warm, damp days, the mass fraction of water vapour is less than 1.5%.

**FIG. 4. CONTROL VOLUME ANALYSIS OF CONDENSATION IN A PARALLEL DUCT.**

Before addressing the problem of predicting the likely occurrence and position of condensation under given operating conditions, it is instructive to assess the effects on the intake flow assuming such an event does occur. For demonstration purposes, a simple model is proposed, namely that the flow is steady, one-dimensional, adiabatic and inviscid in a duct of constant cross-sectional area (see Fig. 4). At inlet to the duct, the stagnation pressure $p_0$, the stagnation temperature $T_0$ and the relative humidity $\phi_0$ are known. During the expansion to a given Mach number $M_1$ upstream of the condensation zone, the flow is assumed to be isentropic and, after crossing the saturation line, to remain in a dry (metastable) state. Downstream of the condensation zone thermodynamic equilibrium is assumed to be re-established. The analysis is rather similar to the Rankine-Hugoniot analysis of a normal shock wave in a parallel duct.

All conditions upstream of the condensation zone (station 1) can be calculated from the specified inlet stagnation state and the assumption that the air and water vapour behave as a mixture of perfect gases. Conditions downstream of the condensation zone (station 2) are computed by application of the conservation equations of mass, momentum and energy.

Let the mass of air per unit mass of mixture be $g$. The mass of water vapour plus liquid per unit mass of mixture is therefore $(1-g)$. Assuming zero velocity slip between gas and liquid droplets, $g$ remains constant throughout the flowfield. The local wetness fraction $y$ is defined as the mass of liquid per unit mass of water vapour plus liquid (not mixture). $y$ represents the local fraction by mass of H$_2$O which is in the liquid-phase. The mass of vapour per unit mass of mixture is $(1-g)(1-y)$ and the mass of liquid per unit mass of mixture is $(1-g)y$. Obviously,

$$g + (1-g)(1-y) + (1-g)y = 1 \quad (1)$$

If the partial densities of the air and water vapour components are $p_g$ and $p_v$ respectively, then the density of the gas-phase is $\rho = p_g + p_v$ and the mixture density $\rho_m$ is (neglecting the volume occupied by the liquid droplets),

$$\rho_m = \frac{\rho}{g + (1-g)(1-y)} \quad (2)$$
If \( h_g, h_v \) and \( h_{w} \) represent the specific enthalpies of the air, water vapour and liquid respectively, the specific enthalpy of the mixture \( h_m \) is given by,

\[
h_m = g h_g + (1-g)h_v - (1-g)y(h_v - h_w) \tag{3}
\]

The air and water vapour components are assumed to behave as perfect gases with partial pressures \( p_g \) and \( p_v \) respectively and hence,

\[
p_g = p_g R_g T
\]
\[
p_v = p_v R_v T \tag{4}
\]

where \( T \) is the temperature of the gas phase and \( R_g \) and \( R_v \) are the specific gas constants of the air and water vapour. By Dalton's law, the mixture pressure is given by \( p = p_g + p_v \), the partial pressure of the liquid droplets being neglected. (The term "total pressure" is avoided as this can be confused with the stagnation pressure.)

The conservation equations are identical in form to those for single-component, single-phase flow. Thus, for parallel duct flow,

\[
\rho_m V_1 = \rho_m V_2
\]
\[
p_1 + \rho_m V_1^2 = p_2 + \rho_m V_2^2 \tag{5}
\]
\[
h_{m1} + \frac{1}{2} V_1^2 = h_{m2} + \frac{1}{2} V_2^2
\]

At station 1 upstream of the condensation zone, the flow is dry and \( y_1 = 0 \). At station 2 thermodynamic equilibrium is assumed and,

\[
p_v(T_2) = p_v(T_2)
\]

where \( p_v(T_2) \) is the saturated vapour pressure at temperature \( T_2 \). It therefore follows that the term \( (h_{v2} - h_{w2}) \) in the expression for the mixture enthalpy is equal to \( h_{fg}(T_2) \), the specific enthalpy of evaporation also evaluated at temperature \( T_2 \). Noting the relationships,

\[
h_{g2} - h_{g1} = c_{pg}(T_2 - T_1)
\]
\[
h_{v2} - h_{v1} = c_{pv}(T_2 - T_1)
\]

where \( c_{pg} \) and \( c_{pv} \) are the isobaric specific heat capacities of air and water vapour respectively, the energy equation can be written,

\[
(g c_{pg} + (1-g)c_{pv})(T_2 - T_1) - (1-g)y_2 h_{fg}(T_2) + \frac{1}{2}(V_2^2 - V_1^2) = 0 \tag{8}
\]

An empirical expression for the saturated vapour pressure of water (with range of validity from \(-50 \degree C\) to \(+50 \degree C\)) is given by Lowe and Ficke and quoted on p.625 of Pruppacher and Klett (1980). \( h_{fg} \) can be obtained in a thermodynamically consistent manner from this equation by application of the Clausius-Clapeyron equation.

Upstream of the condensation zone the mixture is in a nonequilibrium, metastable state. Departures from equilibrium are represented in terms either of the subcooling \( \Delta T \) or the supersaturation ratio \( S \). The subcooling is defined by,

\[
\Delta T = T_S(p_v) - T
\]

where \( T_S(p_v) \) is the saturation temperature corresponding to the local vapour partial pressure \( p_v \) and \( T \) is the actual gas-phase temperature. (For superheated states \( \Delta T < 0 \) and for subcooled states \( \Delta T > 0 \).) The supersaturation ratio is defined by,

\[
S = \frac{p_v}{p_v(T)}
\]

(For superheated states \( S < 1 \) and for subcooled states \( S > 1 \).) The description of metastable states by either \( \Delta T \) or \( S \) is equivalent and depends on personal preference. Downstream of the condensation zone where equilibrium is established, \( \Delta T = 0 \) and \( S = 1 \).

For given inlet stagnation conditions \( (p_0, T_0, \theta_0) \) and upstream Mach number \( M_1 \), the above set of equations completely specifies the downstream state \( (p_2, T_2, y_2, M_2) \) following condensation. An analytical solution is not possible, however, and calculations must be performed by an iterative numerical method.

In general terms, the effect of condensation in a high speed flow is similar to the effect of heat addition to a single-phase flow. Thus, in subsonic flow, condensation causes the flow to accelerate and the Mach number to increase. If the Mach number upstream of the condensation zone is high and appreciable phase-change occurs, the flow may become thermally choked. In this situation, there is no steady flow solution of the conservation equations. Practically, the mass flowrate and the upstream Mach number would both fall until the critical condition for choking was just attained at the very end of the duct.

Under the assumptions of the simple model, the condition for thermal choking corresponds to a Mach number of unity based on the equilibrium speed of sound \( a_e \) in the two-component, two-phase mixture. The derivation of an expression for this sound speed is algebraically rather complicated, but the result is (Smolders et al., 1990),

\[
a_e^2 = \frac{A R_T}{B}
\]

(11)

where,

\[
A = 1 + \frac{g R_g}{(1-g)(1-y)R_v} \left( \frac{R_v T}{h_{fg}} \right) \left( \frac{c_p T}{h_{fg}} \right)
\]
\[
B = 1 - \left( \frac{R_v T}{h_{fg}} \right) \left( \frac{c_p - g R_g}{(1-g)(1-y)c_p} \right)
\]
\[
\left( \frac{R_v T}{h_{fg}} \right) \left( \frac{c_p T}{h_{fg}} \right)
\]

4
and $c_{pw}$ is the specific heat capacity of water. Note that $a_e$ is always less than the frozen speed of sound $a_f$ obtained by neglecting the response of the water content to small pressure disturbances. Typically, $a_e \approx 0.9 a_f$.

At Mach numbers of about 0.8, the flow is very sensitive to condensation. This is easily demonstrated by calculating, for a range of inlet stagnation conditions ($P_0, T_0, \phi_0$), the upstream Mach number $M_1$ such that the downstream flow is just thermally choked, $M_{e2} = V_2/a_2 = 1$. The results are shown in Fig. 5. Evidently, unless the relative humidity and ambient temperature are very low, the effect of condensation in accelerating the flow can be dramatic. For example, for most of the year in the U.K., the temperature is above 10 °C and the relative humidity above 50%. Fig. 5 then shows that condensation to an equilibrium state in a parallel duct will thermally choke any flow at a Mach number exceeding about 0.83. The corresponding increase in flow velocity is of the order of 50 m/s. At relative humidities of 80-90% and ambient temperatures greater than 20 °C, the effects are even more pronounced.

Not all the water vapour condenses during the process as some of the H$_2$O must remain in the form of vapour to satisfy the downstream thermodynamic equilibrium requirement, equation (6). The curves of Fig. 6 correspond to those of Fig. 5 and show the fraction of vapour condensed during the acceleration from the limiting Mach number $M_1$ to the thermally choked condition. By reference to Fig. 3 in conjunction with Fig. 6, it can be seen that a comparatively small precipitation of liquid water (usually constituting less than 1% of the total mixture flowrate) can have a very considerable effect on the flow velocity. This result is attributable to the very high latent heat of H$_2$O and the low specific heat capacity of air.

The calculations presented in Figs. 5 and 6 are highly idealised and it is as well to examine critically the assumptions under which they were obtained.

Firstly, it is assumed that the downstream two-phase flow is in thermodynamic equilibrium at $M_{e2} = 1.0$. Consideration of the non-equilibrium processes occurring in the condensation zone, however, shows that this cannot be the case (Young, 1984). Without presenting details, a study of the flow equations demonstrates that condensation occurring in a parallel duct at Mach numbers less than unity but greater than some critical value (usually about 0.92) displays an unusual instability whereby the heat release, instead of promoting a reversion to equilibrium, causes precisely the opposite effect. (The phenomenon is related to the well-known behaviour of Rayleigh flow where, for Mach numbers in the range $1/y < M < 1.0$, external heat addition to the flow causes a decrease in static temperature.) The upshot is that thermal choking occurs very rapidly but always under non-equilibrium conditions. In the context of the present problem, this would have the effect of reducing the limiting upstream Mach number below the values shown in Fig. 5 (see Young, 1984 for more details).

A second point which must be considered is that condensation may take place at an insufficient rate to cause complete reversion to equilibrium by the end of the duct. Later, it will be shown that, in general, foreign nuclei are most likely responsible for most of the phase change and should these not be present in sufficient concentrations, then the rate of condensation will be reduced. Partial condensation will, of course, still result in an increase in flow velocity but not to the extent suggested by the curves of Fig. 5.

Despite these provisos, the simple model of condensation presented above is very instructive in providing a general quantitative understanding of the flow behaviour.
In the engine test performed with humid air, it was found that condensation occurring in the intake was associated with an increase in fan rotational speed. This result is compatible with the above analysis. Assuming the fan to be operating at, or near, the unique incidence condition, an increase in axial velocity of the flow would need to be accompanied by a corresponding increase in fan blade speed.

In a real engine test, however, it is unlikely that the mass flowrate through the engine would remain the same as when water vapour were completely absent (the assumption implicit in the above analysis). In the presence of condensation, the tendency for the flow to accelerate and its pressure to fall would probably be counteracted by a reduction in mass flowrate entering the engine in order to maintain the static pressure at the engine face-plane approximately constant. Obviously a more complete analysis involving the interaction of the intake system with the whole engine is required in order to predict the precise changes in operating conditions associated with the presence of condensation. This also raises the interesting question of the effect on the fan performance of droplet evaporation due to the static pressure increase within the blading. Evaporation is unlikely to be complete at fan exit because of the small flow transit time, but even partial evaporation would result in interphase heat and mass transfer between the droplets and the gas and may have a significant effect on fan performance by redistributing the flow and changing the position of shock-waves.

STAGNATION PRESSURE LOSS DUE TO CONDENSATION

Because of the interphase transfer of heat and mass, condensation is a thermodynamically irreversible process and results in an increase in entropy. When vapour molecules condense on a liquid droplet, 'latent heat' is initially released at the droplet surface. This heat is then conducted back to the gas-phase where it increases the gas temperature, thus reducing the vapour subcooling and promoting the reversion to equilibrium. For this process to occur, the droplet temperature must be higher than that of the gas-phase and the resulting heat transfer across a finite temperature difference is responsible for the creation of entropy.

For the case of condensation in a parallel duct, the details of the droplet formation and growth processes are of no relevance if the sole objective is to calculate the entropy increase during the reversion from a dry metastable condition to a wet equilibrium state. (In this respect, the analysis is similar to that for a normal shock wave where it is also unnecessary to consider the details of the dissipation processes due to viscosity and heat conduction occurring within the wave itself.) Once conditions upstream and downstream of the condensation zone have been established, the increase in mixture specific entropy $\Delta s$ follows from the expression,

$$\Delta s = \left[ g s g_2 + (1-g)y_2 s v_2 + (1-g)y_2 s w_2 \right] - \left[ g s g_1 + (1-g)s v_1 \right], \quad (12)$$

where $s_g$, $s_v$ and $s_w$ are the specific entropies of air, water vapour and liquid water respectively. Noting that the air and water vapour behave as perfect gases, this may conveniently be written,

$$\Delta s = \left[ g c_p g + (1-g)c_p v \right] \ln \frac{T_2}{T_1} - g R_g \ln \frac{P_{g 2}}{P_{g 1}} - (1-g)R_v \ln \frac{P_{v 2}}{P_{v 1}} - \left(1-g\right)y_2 h_{fg}(T_2) \frac{T_1}{T_2}. \quad (13)$$

Having calculated the entropy increase across the condensation zone, the question now arises as to what is the corresponding stagnation pressure loss and whether or not conventional Pitot probes will record this loss correctly.

There is no difficulty in answering the first part of this question unambiguously. The stagnation pressure is defined as the pressure obtained if the flow is brought to rest adiabatically and reversibly. Downstream of the condensation zone, the two-phase flow is assumed to be in thermodynamic equilibrium. (Difficulties arise if this is not the case and the definition of stagnation pressure must then be re-examined.) A process can be conceived which decelerates the flow (including the droplets) to zero velocity while maintaining negligible interphase velocity slip and, simultaneously, allowing droplet evaporation to occur at such a rate that the temperature difference between the phases is also negligible. The pressure rise associated with this isentropic process is well-defined and easily calculated from the energy and momentum conservation equations. In this way, it is possible to compute a well-defined stagnation pressure loss $\Delta P_o = P_{o 1} - P_{o 2}$ across the condensation zone.

Figures 7 and 8 show curves of $\Delta P_o / P_{o 1}$ for parallel duct flows which, after reversion to equilibrium, are thermally choked. Figure 7 shows the variation of $\Delta P_o / P_{o 1}$ with inlet stagnation relative humidity at a constant inlet stagnation temperature of 20 °C. Figure 8 shows the variation with inlet stagnation temperature for a constant relative humidity of 70%. In both figures, the curves are labelled "true loss" because they represent the actual stagnation pressure loss. The loss increases with both relative humidity and temperature but is surprisingly small, hardly rising above 1% even under extreme conditions.

When a Pitot tube is inserted into a gas-droplet flow, the deceleration of the fluid approaching the tube is rapid and it is unlikely that the equilibrium conditions required for measuring the true stagnation pressure are achieved in practice. The conditions which do exist in the vicinity of the probe depend, among other things, on the droplet size but, apart from a recent analysis of Pitot measurements in pure steam by White (1992), there are few references in the literature to provide guidance for this difficult problem. It is possible, however, to define a limiting case whereby the droplets effectively take no part in
the flow deceleration because they are deflected by the Pitot tube without change in mass, momentum and energy. This "fully-frozen" deceleration of the gas stream would result in the measurement of a much lower stagnation pressure and a considerable over-estimate of the loss as is evident from the relevant curves in Figs. 7 and 8.

Other simple assumptions can also be made. Thus, the curves labelled "semi-frozen (A)" in Figs. 7 and 8 assume that full thermal and velocity equilibrium of the droplets occur during deceleration but that there is insufficient time for phase change so that the liquid mass fraction remains constant. These assumptions may well be quite realistic in practice because the relaxation time associated with phase change is an order of magnitude greater than the relaxation times associated with droplet temperature and velocity slip. The curves labelled "semi-frozen (B)" provide yet another possibility. Here it is assumed that the droplet temperature relaxes instantaneously to its steady-state value but that mass and momentum transfers are frozen.

The analysis of White showed that the pressure recorded by a Pitot tube in a gas-droplet two-phase flow is strongly influenced by the size of the droplets. However, assuming that the droplet diameters are comparatively large (i.e., of the order of 1 \( \mu \text{m} \)), White's analysis would suggest that Pitot tube measurements are likely to be in closer correspondence with the curves labelled "semi-frozen (A)" than those labelled "true loss". This conclusion is, indeed, in accord with the on-site measurements from the Rolls-Royce engine test referred to above, where the recorded loss was of the order of 2 - 3%. Evidently, caution must be exercised in interpreting Pitot measurements in condensing flows because the true stagnation pressure loss may not be as serious as is indicated by the uncorrected measurements.

**HOMOGENEOUS NUCLEATION**

In the previous sections, an analysis of condensation was presented, the assumption being that this would occur somewhere in the parallel intake duct. Further progress in this direction would require much more complex two-dimensional, non-equilibrium calculations. Instead, attention is shifted to predicting the inlet conditions under which phase change may be expected and the approximate position in the duct at which this occurs. Initially, only the possibility of homogeneous nucleation is considered.

Homogeneous (or spontaneous) nucleation occurs in supersaturated pure vapours or mixtures of vapours and inert gases when foreign particles (dust, salt molecules, ions, etc.) are absent. Under these circumstances, the formation and growth of a liquid nucleus is essentially a random event, the probability of which increases with increasing departure from equilibrium as measured by the subcooling \( \Delta T \) or the supersaturation ratio \( S \) [see equations (9) and (10)]. The theory of homogeneous nucleation is well-developed but agreement with experimental measurement is by no means precise even for a substance like water which has formed the basis for a very large number of experimental investigations. A critical discussion of homogeneous nucleation in nozzle expansions of pure steam can be found in Young (1982) and information on nucleation in moist air can be found in Wegener (1966).

The classical theory of homogeneous nucleation is very clearly described by McDonald (1962/63). There, it is shown that the rate of nucleation of liquid droplets per unit volume in a supersaturated vapour is given by,

\[
J = \sqrt{\frac{2\pi}{\pi m^3}} \frac{\rho_v}{\rho_w} \exp \left( -\frac{4\pi \sigma^2}{kT} \right),
\]

(14)
where $\sigma$ is the liquid surface tension, $m$ is the mass of a water molecule and $k$ is Boltzmann's constant. The Kelvin-Helmholtz critical radius $r_*$ is given by,

$$r_* = \frac{2\sigma}{\rho w R_v T \ln(S)} \quad (15)$$

Obviously $J$ is a very strong function of the supersaturation ratio and the nucleation rate changes by several orders of magnitude for quite modest changes in $S$.

Over the years, a number of corrections and amendments to the classical theory have been proposed. Experiments indicate, however, that, in the present context where the water vapour content is low, equation (14) is probably a fairly accurate representation of reality. Difficulties in application arise because the temperature in the engine intake upstream of condensation falls below 0 °C, but in this situation the relevant saturated vapour pressure is that of subcooled liquid water rather than solid ice. (Although the equilibrium condensed phase at temperatures below the triple point is ice, Ostwald's law of stages and experimental evidence suggests that it is the liquid rather than the solid phase which is first nucleated.)

When first nucleated, droplets are assumed to be at the critical radius $r_*$. At high supersaturation ratios such as occur during rapid expansions, $r_*$ is very small indeed and may represent a liquid cluster containing less than 50 molecules. Following nucleation, however, the clusters immediately start to grow to macroscopic size by the condensation of vapour from the gas-phase. Equations describing the rate of growth are therefore required because it is the conduction to the gas-phase of the latent heat liberated by condensation at the droplet surface which causes the reduction in subcooling and the corresponding decrease in the nucleation rate as the flow reverts to equilibrium. The droplet growth equations have been the focus of many theoretical studies and the subject is too complex to discuss here. Suffice it to say that the equations used for the present calculations are those derived by Young (1992a) which provide an accurate representation of droplet growth rates for a wide range of water vapour concentrations and droplet radii.

In order to predict the onset of homogeneous condensation in a rapid expansion, it is necessary to integrate the nucleation and droplet growth equations in conjunction with differential forms of the gas dynamic conservation equations. Calculations of this type in one-dimensional nozzles have been performed in fairly small nozzles (10 - 50 cm in length) with correspondence high rates of expansion. Under these circumstances, for high inlet stagnation relative humidities, an expansion to slightly supersonic Mach numbers is required before significant quantities of water are produced by homogeneous nucleation. Decreasing the inlet relative humidity has the effect of increasing the Mach number at the

computed dry intake pressure distribution as shown in Figs. 1 and 2. This requires a very much simpler, essentially one-dimensional, calculation and, although exact quantitative results cannot be expected, experience has shown that the technique is very useful in indicating the correct trends.

The input data for the calculations are therefore the pressure distributions of Fig. 2 rather than the geometrical co-ordinates of the engine intake. The mass continuity equation is discarded and the differential forms of the momentum and energy equations (5) combined to give the thermodynamic form,

$$\frac{D\rho m}{Dt} - \frac{1}{\rho m} \frac{D\rho}{Dt} = 0 \quad (16)$$

It is interesting to note that (in the absence of velocity slip), equation (16) is valid for two-phase as well as for single-phase flows. In single-phase flow, equation (16) implies that the entropy of a fluid particle remains constant. This is not the case, however, when two phases in thermal disequilibrium are present. (It is a standard result that, for non-equilibrium two-phase flow the left hand side of equation (16) cannot be equated to $D\rho m/Dt$.) Equation (16) is therefore not incompatible with the production of entropy due to irreversible condensation as described previously.

If the pressure variation is specified, the thermodynamics of the problem are effectively uncoupled from the fluid dynamical influences (in the sense that the flow velocity does not enter the equations) and equation (16) in conjunction with equation (14) and the droplet growth equations can be integrated in step-wise fashion in the flow direction. A detailed description of how this can be achieved for nucleation in expansions of pure steam can be found in Guha and Young (1991) and Young (1992b). Calculations for moist air are similar and the details will not be presented here.

For given inlet stagnation conditions ($p_0 , T_0 , \phi_0$) and specified pressure distribution, the calculations predict the variation along the flow path of the subcooling $\Delta T$, the wetness fraction $\gamma$ and the droplet size distribution. The position of maximum subcooling (which coincides almost exactly with the maximum homogeneous nucleation rate $J$) is called the "Wilson Point". Immediately after the Wilson Point, the subcooling and nucleation rate fall rapidly as the condensational growth of existing droplets and release of "latent heat" promote the reversion to equilibrium. Theory and experiment show that the mean droplet diameter downstream of the main condensation zone (i.e., after reversion to equilibrium) is related to the maximum subcooling (at the Wilson Point) and both are a strong function of the rate of expansion in the nucleation zone just upstream of the Wilson Point. Most experiments reported in the literature have been performed in fairly small nozzles (10 - 50 cm in length) with correspondingly high rates of expansion. Under these circumstances, for high inlet stagnation relative humidities, an expansion to slightly supersonic Mach numbers is required before significant quantities of water are produced by homogeneous nucleation. Decreasing the inlet relative humidity has the effect of increasing the Mach number at the
Wilson Point to higher values and displacing the condensation zone well into the supersonic part of the flow. The corresponding mean droplet diameters are very small, usually of the order of 0.1 μm or even less.

As shown in Fig. 2, the rate of expansion along the streamline adjacent to the cowling of the full-scale intake is comparatively high. In this region, homogeneous nucleation is therefore expected to occur at much the same Mach numbers as in smaller laboratory nozzles. The results of the numerical calculations confirm this expectation. Figure 9 shows the position of the Wilson Point on the cowling wall streamline in terms of the inlet stagnation temperature and relative humidity. (The abscissa scale of Fig. 9 has the same (arbitrary) origin as Fig. 2 but is greatly expanded because nucleation only occurs in a very narrow region close to the position of minimum pressure.) Taking, as an example, the calculations for an inlet stagnation temperature of 30 °C, it can be seen that, for a relative humidity of 100%, the Wilson Point occurs at a streamwise distance of 46 mm which corresponds to a Mach number of 0.96. (The Mach number corresponding to each streamwise position can be read from the scale at the top of the graph.) Decreasing the relative humidity displaces the Wilson Point to higher Mach numbers until, at \( \phi_r = 38\% \) (for \( T_0 = 30^\circ C \)), it is situated at a streamwise position of 140 mm which corresponds to the point of minimum pressure in Fig. 2. (The Mach number at this point is about 1.1.) At lower values of \( \phi_r \), homogeneous nucleation is inhibited because the maximum nucleation rate (which occurs at the minimum pressure point) falls rapidly with decrease in inlet relative humidity.

The boundary between wet and dry operation (in respect of homogeneous nucleation) therefore corresponds (at least approximately) to those inlet conditions such that the Wilson Point coincides with the point of minimum pressure on the streamline. Figure 10 shows this boundary (in terms of the inlet stagnation temperature and relative humidity) for the wall pressure distribution of Fig. 2. Other intake geometries would, of course, result in a different boundary between wet and dry operation. In particular, homogeneous nucleation may be inhibited completely if the design is such that the peak Mach number is restricted to subsonic values less than about 0.9.

The numerical calculations provide a complete analysis of the reversion to equilibrium but the details will not be presented here. However, it is of interest to note that, for all the cases shown in Fig. 9, the Wilson Point subcooling was in the range 33 - 39 °C and the mean droplet diameter far downstream of the condensation zone was in the range 0.1 - 0.3 μm. (Decreasing the inlet relative humidity at constant stagnation temperature results in a slight decrease in Wilson subcooling and a slight increase in droplet diameter. Decreasing the stagnation temperature at constant relative humidity has the reverse effect.)

Turning now to the pressure distribution along the centre-line of the intake, it can be seen from Fig. 2 that the Mach number is monotonically increasing and never exceeds a value of 0.8. At this Mach number, the subcooling does not exceed 27 °C (for the range of inlet temperatures considered) and the corresponding homogeneous nucleation rates are low, suggesting that condensation by this method is negligible. It is important to recall, however, that the rate of expansion along the centre-line is much lower than along the wall streamline and also there is a very long time available for droplet growth in the straight section of the duct.

Nevertheless, the numerical calculations show that significant condensation by homogeneous nucleation is unlikely except for inlet stagnation temperatures approaching 30 °C and relative humidities near 100%. Figure 11 shows the
wetness fraction at the engine face-plane normalised by the local equilibrium value corresponding to complete reversion. For inlet temperatures below 20 °C, the water content of the flow is effectively zero for all values of φ. Even for an inlet temperature of 30 °C, it is only when the relative humidity exceeds 90% that significant (although incomplete) condensation occurs. In these circumstances, it is the very long intake duct which makes homogeneous nucleation possible because, although the nucleation rate is low, the residence time of 30 ms provides an extended opportunity for droplet growth. Indeed, droplets formed under these conditions have diameters of the order of 3 μm (compared with 0.1 - 0.3 μm for droplets nucleated near the wall). Fig. 10 shows the boundary between wet and dry operation for the centre-line streamline and visually underlines the conclusion that homogeneous nucleation is unlikely to be responsible for significant water precipitation in the core of the flow, except under extreme conditions.

In summary, it has been shown that condensation due to homogeneous nucleation can be expected to occur in a narrow annulus adjacent to the cowling wall near the lip of the intake unless the relative humidity is less than about 50% (dependent on the inlet temperature). Homogeneous nucleation in the main core of the flow is unlikely except for a combination of very high temperature, relative humidity and Mach number.

HETEROGENEOUS NUCLEATION

If the concentration of foreign particles in the atmosphere is high, heterogeneous nucleation and droplet growth may be responsible for condensation in sufficient quantities to affect the flow behaviour. There now exists a considerable bank of data concerning atmospheric particle concentration and size distribution (Pruppacher and Klett, 1980) and it is evident that particle concentrations are very much higher in the vicinity of large industrial conurbations than in isolated rural areas. Foreign particles act as condensation nuclei (just as in cloud formation) and, if the concentration is sufficiently large, the resulting precipitation of water can have a significant effect on the flow. Cloud condensation nuclei (CCN) lie in the size range 0.1 - 5.0 μm and can be "activated" (i.e., start to grow by vapour condensation) at subcoolings very much lower than those required for homogeneous nucleation. (However, although it is unusual to encounter subcoolings greater than 1 °C in the atmosphere, it should be remembered that the cooling rates associated with cloud formation are orders of magnitude lower than those found in nozzle-type flows.)

Figure 12 shows the size distribution of "large" atmospheric particles actually measured by Rolls-Royce at Hucknall (the site of the jet engine test involving humid air). ("Large" particles is a well defined category and refers to a range of diameters from 0.2 - 2.0 μm.) The distribution and overall concentration are typical of measurements made near industrial sites. In all, six measurements were made, the total recorded particle concentration ranging from 4000 to 8000 particles per cm³. Figure 12 shows a typical distribution with a total concentration of 6210 particles per cm³ and a mass mean diameter of 0.19 μm. It is important to note, however, that the particle counter was unable to record particles with diameters smaller than 0.1 μm and that these unrecorded "Aitken" nuclei are likely to be present in large concentrations at ground level (Pruppacher and Klett, 1980, p.204). Although Aitken particles are too small to act as nuclei in normal cloud formation, they can be activated by the expansion to high subcoolings in a jet engine intake. Calculations based on measured concentrations of "large particles" therefore tend to underestimate the quantities of water condensed, possibly by quite a large factor.

The process of heterogeneous nucleation, unlike that of homogeneous nucleation, is not well understood and, at
present, no satisfactory theory exists. Atmospheric particles are not spherical and their composition varies, some being water soluble and others not. In order to make progress, therefore, a very simple mechanism for nucleation is postulated which, it must be admitted, is unlikely to model the real physical processes with any great accuracy. Thus, it is assumed that all particles are spherical and insoluble in water, and start to grow as spherical water droplets with a solid core when the Kelvin-Helmholtz critical radius \( r_\text{c} \) [defined by equation (15)] becomes marginally smaller than the actual particle radius. For an inlet stagnation temperature of 20 °C and various relative humidities, Fig. 13 shows the Mach number required to activate atmospheric particles of different diameters. It is evident that, for the measured size distribution of Fig. 12, all the particles present (and many more Aitken nuclei) will be activated in an expansion to a Mach number of 0.8 unless the stagnation relative humidity is very low indeed.

Equation (15) implies that particles of diameter 0.1 μm are activated at very low subcoolings of about 0.2 °C. As an approximate ‘rule of thumb’, it can therefore be assumed that all the larger atmospheric particles become activated if the water vapour becomes dry saturated during an expansion. Figure 14 shows the range of inlet conditions and Mach numbers required to activate these particles under the above assumption. Obviously, unless the relative humidity is very low, most atmospheric particles will be capable of acting as nuclei for vapour condensation during expansion in an engine intake.

However, even if all the foreign particles present become activated, this does not necessarily imply that sufficient water vapour will have condensed by the engine face-plane to appreciably affect the flow behaviour. The precise quantity condensed will also depend on the total concentration of particles and the rate of condensation as specified by the droplet growth equations.

In order to investigate these factors, calculations were performed adopting the model of heterogeneous nucleation described above. For simplicity, it was assumed that the particles were monodispersed with a diameter of 0.2 μm, the mass mean diameter of the measured distribution. (In fact, the results are almost independent of the size distribution so long as most of the particles are larger than about 0.01μm diameter.) Calculations were performed for a range of particle concentrations and inlet stagnation conditions. Figure 15 shows the wetness fraction at the engine face-plane (normalised by the equilibrium value corresponding to complete reversion) for an inlet stagnation temperature of 20 °C and atmospheric particle concentrations in the range 5x10³ - 1x10⁵ per cm³.

The results of Fig. 15 confirm that, below a relative humidity of about 15 %, the flow at the engine-face plane has no water
content. This is simply because, in an expansion to a Mach number of 0.8, the water vapour remains unsaturated (S < 1). For relative humidities exceeding this threshold value, the fraction of water vapour condensed depends strongly on the concentration of foreign particles present. The on-site measured concentration of Fig. 12 corresponds approximately to the curve labelled 5000 particles per cm³ in Fig. 15. At this concentration the calculations indicate that only about 10% of the equilibrium wetness fraction will have condensed by the end of the intake duct. However, as noted above, the concentration of Aitken nuclei is very much higher than the concentration of CCN (perhaps by a factor of 10 or more) and the curves labelled 5x10⁴ or 1x10⁵ are probably more representative of the actual concentration of particles which become activated during the expansion. Assuming this to be the case, then very significant quantities of water will be present at the engine face-plane for relative humidities exceeding, say, 25% (at T₀ = 20 °C) and marked changes in the flow behaviour should therefore be expected.

As plotted in terms of (y/yₑq), the curves of Fig. 15 are largely independent of the inlet stagnation temperature. However, for a given relative humidity, the vapour content of the atmosphere falls with temperature (see Fig. 3) and hence the actual mass fraction of condensed water at the engine face-plane also decreases with temperature. The effect on the flow behaviour is therefore not so pronounced at lower temperatures.

Because atmospheric particle concentrations are comparatively low (in comparison with droplet number densities resulting from homogeneous condensation), heterogeneous nucleation usually results in droplet sizes which are larger than those associated with homogeneous nucleation. For the calculations shown in Fig. 15, the droplet diameters at the engine face-plane ranged from 1 to 7 µm.

Figure 16 shows the growth of the liquid phase due to heterogeneous condensation as a function of distance along the centre-line streamline for an arbitrary inlet condition of T₀ = 20 °C, φ₀ = 70%. The importance of duct length is now very evident. For short inlets associated with wing-mounted engines, the time available for droplet growth is insufficient for significant condensation to occur and the flow is unlikely to be seriously affected by the presence of foreign nuclei. It is only with the very long intake ducts associated with tail-mounting or purpose-built engine test cells that serious problems are likely to occur during stationary operation.

CONCLUSIONS

A theoretical and computational investigation of condensation in jet engine intake ducts has led to the following conclusions concerning stationary operation:

1. Homogeneous nucleation of water vapour is unlikely to occur in the core flow of the intake (at Mach numbers of 0.8) except for extreme conditions of very high inlet temperature and relative humidity. The converse is true, however, in a narrow annulus adjacent to the intake cowling where the high curvature of the streamlines results in an over-expansion to Mach numbers of about 1.0–1.1. Homogeneous nucleation will occur here for relative humidities exceeding 40–70%, the threshold value depending on the inlet stagnation temperature. Nucleation in this region is independent of the length (but not the geometry) of the intake duct and may, therefore, occur in short intakes designed for wing-mounted engines as well as in long ducts designed for tail-mounting. These conclusions have been substantiated by visual observation during stationary engine tests where condensation is often observed to occur in the region described above.

2. Heterogeneous condensation on atmospheric particles will occur whenever the inlet stagnation conditions are such that the water vapour becomes slightly supersaturated during the expansion to the engine face-plane. Particles exceeding 0.1 µm diameter (cloud condensation nuclei, CCN) become activated at subcoolings of 0.2 °C or less. Smaller Aitken nuclei require higher subcoolings but, unless the relative humidity is very low, it is expected that most atmospheric particles will be capable of acting as centres of condensation in an expansion to a high subsonic Mach number. The fraction of water condensed (and hence the effect on the flow) depends strongly on the total particle concentration and the time available for droplet growth (i.e., the duct length). For tail-mounted engines with very long intake ducts, heterogeneous condensation can be substantial for particle concentrations of the order of those found near industrialised areas. For wing-mounted engines, the intake is short and heterogeneous condensation is less important.

3. The actual mass fraction of vapour condensed in an
The effect of condensation at subsonic Mach numbers is to accelerate the flow. The effect can be dramatic at dry air Mach numbers of 0.8, when reversion to equilibrium in a parallel duct may be sufficient to thermally choke the flow (assuming the mass flowrate to remain constant). In practice, the mass flowrate through the engine may not remain constant but a more complete analysis involving the flow through the fan and bypass duct (where evaporation will occur) is required to predict the exact changes in engine operating conditions. However, an increased flow velocity approaching the engine is consistent with experimental on-site measurements which detected an increase in fan rotational speed (required to maintain the unique incidence condition at blade inlet) when condensation occurred.

5. Because of the interphase temperature difference, condensation is a thermodynamically irreversible process and is associated with an increase in entropy and decrease in stagnation pressure. Calculations show that the loss in stagnation pressure is small, seldom exceeding 1%. Conventional Pitot probe measurements may not record this loss correctly because the rapidly decelerating two-phase flow in the vicinity of the probe can deviate markedly from equilibrium. The flow behaviour in this region is complex and difficult to predict but a simplified analysis showed that stagnation pressure losses computed from uncorrected Pitot measurements in the usual way could overestimate the true values by factors of 2 or 3. On-site measurements obtained during engine testing were in agreement with this prediction.

The analysis presented in this paper has demonstrated the importance of various aspects of condensation theory as applied to the stationary operation of jet engines, especially those fitted with very long intake ducts. The only way to avoid these complications during engine testing is either to locate the test site in a hot, dry region where the relative humidity is always low or to confine the test programme to the winter months when ambient temperatures are low. Alternatively, the analysis may be viewed as providing the inspiration for the further investigation of some most unusual and interesting phenomena which have a not insignificant effect on engine performance under certain operating conditions.

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