A PRACTICAL METHOD FOR PREDICTING THE WINDMILLING CHARACTERISTICS OF SIMPLE TURBO JET ENGINES

Min Su Choi†, Jin Shik Lim‡, and Yong Shik Hong††

ABSTRACT
A practical method for predicting the windmilling performance of simple turbojet engines in flight has been developed. The method incorporates the available loss correlations and analyses to estimate the major engine component performance and requires only basic geometric data. For a given flight Mach number and ambient conditions the steady-state windmilling performance is obtained by equating the compressor input torque to the turbine output torque. The transient performance is obtained from the excess turbine output torque. The present method's predictions are compared to the available windmilling test data from a typical small simple turbojet engine. They show good agreement over a wide range of flight conditions. Thus, this method can be used during the preliminary design stage of an engine development when the detailed hardware geometry and the component performance data are not yet available.

NOMENCLATURE

- $A$: inflow or outflow path annulus area
- $A_i$: inlet area
- $A_N$: nozzle area
- $a$: distance from the leading edge to the maximum camber location of the airfoil
- $C$: absolute velocity of air
- $C_l$: lift coefficient
- $C_v$: chord
- $C_d$: drag coefficient
- $C_{aw}$: annulus wall drag coefficient
- $C_{sw}$: profile drag coefficient
- $C_{saw}$: secondary loss drag coefficient
- $C_{daw}$: secondary drag coefficient
- $C_e$: constant in the force equation
- $h$: altitude
- $H$: blade height
- $i$: incidence angle
- $i_{opt}$: optimum incidence angle
- $I_p$: polar moment of inertia
- $J$: mechanical equivalent of heat
- $k_r$: kinetic energy loss coefficient for rotor
- $k_s$: kinetic energy loss coefficient for stator
- $L_r$: rotor energy loss
- $L_s$: stator energy loss
- $L_i$: incidence loss
- $M_a$: flight Mach number
- $m_a$: air mass flow rate
- $N$: rotational rpm
- $P$: pressure
- $P_{s}$: Mach number
- $r_H$: hub radius
- $r_m$: mean radius
- $r_t$: tip radius
- $s$: blade pitch
- $T$: temperature
- $t$: time
- $l_e$: tip clearance
- $T_o$: torque
- $U$: rotational speed
- $V$: flight speed
- $W$: relative velocity of air
- $\beta$: relative air angle

† Agency for Defense Development, Daejon, Korea
‡ Professor, Department of Aerospace Engineering,
‡‡ Dinha University 253, Yong-Hyung Dong, Nam-Gu, Incheon 402-751, Korea.
tests of several turbojet engines under various windmilling conditions and developed some generalized correlations. This correlation method is also applicable to engines similar to the ones investigated.

A theoretical study of windmilling turbojet engine was made by Shou(8). Generalized curves which characterize the windmilling performance were obtained from experimental data of several turbojet engines. Again, the accuracy of the method is limited to those engines similar to the reference engines.

The objective of the present work was to investigate the physical mechanism of windmilling and to develop a practical prediction method which can be used in the early stages of engine development where few component test data are available.

2. CHARACTERISTICS OF WINDMILLING OPERATION

Windmilling of an engine results from the high dynamic pressure ambient air going through the engine, rotating the engine rotor. The rotational speed and the direction depend on the vector sum of the frictional torque and the aerodynamic torque which act on each rotor. The mass flow and the flow velocity through the engine depend on the flow area and the over-all pressure drop across the engine.

During the transient windmilling phase the axial compressor goes through turbine mode, frictional duct mode, and finally compressor mode as its speed increases. As shown in Fig.1 the compressor acts like a turbine when rotational speed is close to zero. As the speed increases to the point where tangential velocity component, \( C_w \), at inlet become equal to that at the exit, no torque is acted on the compressor. Therefore, the compressor behaves like a frictional duct. However, at high rpm, the compressor regains its compressor mode. On the other hand, the air flow always expands and continues to deliver energy to the turbine. Unlike the fired engine operation no heat transfer is involved and the flow through the engine can be considered adiabatic. Therefore, although the total temperature varies through the compressor and the turbine, the total temperature at the engine exit remains same as that at the inlet.

3. CALCULATION PROCEDURE

The numerical calculation procedure is composed of two iteration loops (Figure 2). The inner loop is to determine the windmilling operation conditions of each component satisfying mass flow balance. But this operating condition means a torque imbalance between the compressor and the turbine. So, the final operating condition of each component can be found through the outer loop in which the excess torque of a turbine is calculated at each iteration step.

4. LOSS THROUGH AXIAL COMPRESSORS

According to Horlock(9) and Dixon(10), the stagnation pressure loss through axial compressor blade rows arises from three sources - the profile drag, the annulus wall drag, and the secondary flow effects. The profile drag coefficient \( C_{dp} \) can be obtained from Fig. 3 for various incidence angles \( \alpha' \). The corresponding relative deflection \( \varepsilon/\varepsilon' \) can also be obtained from
Figure 2: Calculation process flow chart.

Figure 3: Off-design performance of a compressor cascade (9).

Figure 4: Effect of tip clearance fraction and degree of reaction on tip clearance loss (11).

Figure 5: Effects of flight Mach number and altitude on steady state windmilling speed.
the same figure. The annulus wall drag coefficient $C_{Da}$ and the secondary flow loss coefficient $C_{Dp}$ may be estimated from Equations (1) and (2), respectively (Refs. (9), (10), (15)).

$$C_{Da} = 0.02 \ (s/H) \quad (1)$$

$$C_{Dp} = 0.018 \ C_L^2 \quad (2)$$

$C_{Da}$ is assumed to include a component for minimum tip clearance loss (Equation (2)). The sum of the three loss coefficients $C_D = C_{Da} + C_{Dp} + C_{Df}$ gives the total pressure loss $\Delta P_{T}$ and the stage efficiency $\eta_{stage}$ (Steps 13&15 in Appendix A).

For a multi-stage compressor the performance calculation is made by stage stacking techniques. Since the inlet and exit annulus area, the mean station diameter, the blade angles, and the velocity diagram of each stage are given in the engine design stage, the efficiency of each stage computed above, is incorporated to obtain the overall compressor efficiency. The mean work done factor (Refs. (11), (19), (15)) is applied in the analysis to account for the performance deterioration due to velocity profile distortion and the boundary layer development as the flow proceeds downstream in a multi-stage compressor.

5. LOSS THROUGH AXIAL TURBINES

The turbine stage efficiency may be calculated from the estimated values of the stator energy loss, $L_s$, the rotor energy loss, $L_r$, and the incidence loss $L_i$ (Refs. (10), (11)).

$$L_s = k_s \left( \frac{C_{Da} + C_{Dp}}{2 \omega J} \right) \quad \text{Stator} \quad (3)$$

$$L_r = k_r \left( \frac{W_{in} + W_{out}}{2 \omega J} \right) \quad \text{Rotor} \quad (4)$$

Coefficients $k_s$ and $k_r$ represent the profile loss at the design point and are obtained from the Soderberg correlation (Refs. (10), (11)).

$$L_i = \frac{C_{Da}}{2 \omega J} [1 - \cos \gamma (i - i_{ext})] \quad (5)$$

$n = 2$ (positive incidence) \quad $i_{ext} = -4^\circ$

$n = 3$ (minus incidence)

The tip clearance loss in terms of $\eta / \eta_0$ may be obtained from Fig. 4 for different tip clearance fraction and degree of reaction. Although the windmilling state is far from the design point the above loss relationships based on the design point data appear to be accurate (Ref. 11).

6. OTHER LOSSES

The air inlet adiabatic efficiency $\eta_i$ and the engine exhaust nozzle adiabatic efficiency $\eta_N$ have been assumed to be constant. The total pressure loss of the burner has a significant effect on the results and it is advisable to use the test data if available. The total temperature of the air stream remains constant in the burner. The mechanical efficiency of turbines has been assumed to be 0.95 and the accessory drive power is assumed to be proportional to the square of the rotational speed (Refs. (13), (14)). It should be pointed out here that widely used loss models were used for this study. However, other loss models can be incorporated into this method as well.

7. MASS FLOW DETERMINATION

The mass flow rate which goes through the engine depends on the axial velocity $C_a$. Thus, the mass flow rate influences the velocity diagram significantly: the incidence angle and the magnitude of the losses vary accordingly.

The ambient condition $P_s$, $T_s$, and flight Mach number $Ma$ give the engine inlet total condition $P_{T1}$ and $T_{T1}$. An initial mass flow is assumed from Eq. (6).

$$m_{in} = \rho \ V \ A_i \quad (6)$$

The losses through compressor, turbine and other components are calculated. Using the estimated turbine exit $P_T$ and $T_T$ as the nozzle inlet condition, the nozzle mass flow is calculated and compared with the guessed value. The correct mass flow is obtained by iteration as shown in Fig. 2.

8. TRANSIENT AND STEADY STATE WINDMILLING ANALYSIS

The compressor and turbine torques are calculated from the velocity diagram for the selected rotational speed. The transient windmilling performance is determined from the relationship between compressor and turbine torques.

When the vector sum of these torques becomes zero a steady state windmilling condition is reached and the rotational speed remains constant. The excess turbine torque is expressed by the following relations (Refs. (15), (16)).

$$T_e = T_{e\_turbine} - T_{e\_compressor} - T_{e\_acc} \quad (7)$$

The rotor acceleration is then calculated from the following Eq.(8).

$$T_e = I_s \ \frac{d\omega}{dt} = 2\pi I_s \ \frac{dN}{dt} \quad (8)$$

Calculation steps are shown in detail in Appendix A and sample calculation results are shown in Appendix B.

9. MODEL PREDICTIONS & SENSITIVITY ANALYSIS

A Microturbo TRI 60 engine was chosen as the test engine for this study. TRI 60 is a 900-lb thrust class simple turbojet engine having a three-stage axial compressor, a single-stage axial turbine, and a nozzle with 13.5 lbm/s maximum ground static mass flow rate. Approximate geometrical data at the mid-station were obtained from the hardware component measurement and available geometrical information.

A parametric study was carried out to examine the sensitivity of model predictions on various turbine geometric parameters - pitch/chord ratio, turbine blade angles, intake area, turbine inlet area, and exhaust nozzle area. Table 1 shows the model prediction variations from the baseline predictions for ±1% variation in intake area, turbine blade angles, and exhaust nozzle area. When one parameter was varied all of the other values were held at the baseline (i.e. test engine) values. Overall, the model predictions are
insensitive to small variations in turbine geometry. Therefore, during the preliminary design stage when only approximate geometric data are available, this method can still be used to yield meaningful results. The parametric study actually examined ±10% variation from the baseline configuration.

<table>
<thead>
<tr>
<th>RPM</th>
<th>Intake Area</th>
<th>Turbine Blade Angle</th>
<th>Exhaust Nozzle Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>±0.03%</td>
<td>±0.09%</td>
<td>±0.22%</td>
<td></td>
</tr>
<tr>
<td>Mass Flow Rate</td>
<td>±0.02%</td>
<td>±0.94%</td>
<td>±0.22%</td>
</tr>
<tr>
<td>Compressor Pressure Ratio</td>
<td>±0.02%</td>
<td>±0.6%</td>
<td>±0.06%</td>
</tr>
<tr>
<td>Time to Reach the Steady-State</td>
<td>0.0%</td>
<td>±0.35%</td>
<td>±0.09%</td>
</tr>
</tbody>
</table>

Table 1. Results of parametric sensitivity analysis. (TRI 60, Sea level, International Standard Atmosphere, Ma=0.7, for each 1% variation from baseline geometric data).

10. COMPARISON OF THE ANALYSIS WITH DATA

The model's predictions are compared with the windmilling test data for a TRI 60 engine from Ref. 12. (Figs. 5, 6, and 7). Fig.5 shows steady-state windmilling speed against flight Mach number and altitude. Present method predicts that the steady-state windmilling speed decreases as the flight altitude increases. The altitude data for the TRI 60 engine were not available at the time of this analysis. However, other experimental data (Refs (6), (7)) confirm the trend of the present analysis. Except in the very low flight Mach number region the agreement is satisfactory. Effects of flight Mach number and altitude on the engine mass flow rate is shown in Fig. 6. The agreement between the present analysis and the data is very good for most of the ranges investigated.

Fig. 7 shows the transient windmilling speeds as a function of flight Mach number. The experimental data were available only for a flight Mach number of 0.5. The close agreement of the present analysis with the data at Ma = 0.5 suggests that similar agreement may be expected for other conditions. As the altitude increases, the inertia and mechanical friction of the rotating parts remain essentially constant while the ram energy of the incoming air decreases. Therefore, the time required to reach the steady state windmilling speed is greater at higher altitudes. From Fig. 7 the time required to reach the ignition speed of TRI 60 (4,700-5,300 rpm) and the flight Mach number limit for ignition can be determined.

11. CONCLUSIONS

A practical method of predicting windmilling characteristics of simple turbojet engines has been developed. The loss analysis for each major component, based on the available cascade data correlation, is carried out. The relationship between the compressor torque and the turbine torque determines the windmilling performance. Comparison of the results of the present method with the available TRI 60 turbojet engine test data has shown a good agreement in all of the major windmilling performance parameters.

Present method requires geometric data but does not require any experimental data. Furthermore, approximate geometric values yield meaningful windmilling predictions, and the method is sufficiently flexible to accommodate incorporation of different loss models. Thus, this method can be conveniently used in the early stages of turbojet development.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{compressor_diagram.png}
\caption{Three different functions of compressor under windmilling condition.}
\end{figure}
REFERENCES


Appendix A
1. Input values
   \( \eta, \eta_m, \eta_N, \lambda, A, \beta, \alpha, \beta' \), pitch, chord, \( n_m, \gamma, \beta' \), \( \beta'' \).

Flight condition(N_m), Ambient condition(P_a, T_a)

2. Calculation of ram effect:

\[
P_{n} = \left[ 1 + \eta_c \frac{\gamma - 1}{2} M_a^2 \right]^{\gamma - 1}
\]

\[
T_{n} = \left[ 1 + \frac{\gamma - 1}{2} M_a^2 \right]^{\gamma - 1}
\]

3. Assume initial rpm(N) and mass flow rate(Eq. 5)

Compressor

Calculate the following(Step 4 - Step 28):

4. H, \( \gamma_c, \beta \), \( \beta' \), i, C, T, P, \( \rho, C_s \)

5. Nominal deviation:

\[
\delta' = \frac{500A}{500 - \delta' c} [0.22(\frac{2a}{c})^3 \beta' + 0.25]
\]

6. Nominal outlet angle: \( \beta'' = \beta' + \delta' \)

7. Nominal incidence: \( i = \beta'' - \epsilon' - \beta' \)

8. Find \( \epsilon' \), \( \epsilon '' \), \( C_{b0} \) from Fig. 2

9. \( \beta' = \beta - \epsilon' \), \( \tan \beta' = \frac{\tan \beta + \tan \beta'}{2} \)

10. Lift coefficient: \( C_L = 2(\gamma_c)\cos \beta'' (\tan \beta' - \tan \beta'') \)
11. $C_{D_{0}}$, $C_{D_{1}}$ from Eqs. (1), (2).
12. Drag coefficient: $C_{D} = C_{D_{0}} + C_{D_{1}} + C_{D_{2}}$
13. Total pressure loss: $\Delta P_{T} = \frac{1}{2} \rho \frac{W_{1}^{2}}{s/c} \cos^{2} \beta_{1} \frac{\cos^{2} \beta_{1}}{\cos^{2} \beta_{m}}$
14. $dP_{\text{anomagic}} = \frac{1}{2} \rho \frac{W_{1}^{2}}{s/c} \tan^{2} \beta_{1} - \tan^{2} \beta_{2} \tan^{2} \beta_{1} \frac{1}{1 + \tan^{2} \beta_{1}}$
15. Stage efficiency: $\eta_{\text{stage}} = 1 - \left( \frac{\Delta P_{T}}{dP_{\text{anomagic}}} \right)$

APPENDIX B

Sample calculation results are presented for the TRI60 engine in sea level flight at Mach number of 0.5 with International Standard Atmosphere. The input values are given in Table 2. One mass flow rate iteration (Steps 4-28) at the start of the windmilling process (N=0) is shown in Table 3; though initially assumed to be 3.511, the mass flow rate will converge to a lower value. Table 4 shows compressor and turbine torque calculation steps, after mass flow rate convergence, for N = 923. Since there is excess turbine torque, N is will be increased (Eq. (8)), and the entire calculation process (Figure 2) will be repeated for the new $N$.

<table>
<thead>
<tr>
<th>INPUT DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine Geometry Data</td>
</tr>
<tr>
<td>$P_{e}$</td>
</tr>
<tr>
<td>$T_{e}$</td>
</tr>
<tr>
<td>$M_{a}$</td>
</tr>
</tbody>
</table>

Table 2. Input data (TRI60 engine, $Ma=0.5$, Sea level, International Standard Atmosphere).

<table>
<thead>
<tr>
<th>Station</th>
<th>Step</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 3</td>
<td>$N = 0$</td>
<td>$m_{o} = 3.511$</td>
</tr>
<tr>
<td>Step 7</td>
<td>$i^{*} = 3.045$</td>
<td></td>
</tr>
<tr>
<td>Step 8</td>
<td>$\varepsilon = 0.2395$</td>
<td></td>
</tr>
<tr>
<td>Step 9</td>
<td>$\beta_{1} = 8.6271$</td>
<td></td>
</tr>
<tr>
<td>Step 12</td>
<td>$C_{o} = 0.0763$</td>
<td></td>
</tr>
<tr>
<td>Step 13</td>
<td>$\Delta P_{T} = 0.0278$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Mass flow rate iteration (TRI60 Engine, $Ma = 0.5$, Sea level, International Standard Atmosphere).
### RESULTS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$ (rpm)</td>
<td>923</td>
</tr>
<tr>
<td>$\dot{m}_e$ (lbm/sec)</td>
<td>2.755</td>
</tr>
<tr>
<td>$t$ (sec)</td>
<td>0.388</td>
</tr>
<tr>
<td>$T_\tau$ ($^\circ$R)/$P_\tau$ (psia)</td>
<td></td>
</tr>
<tr>
<td>$T_\tau$=546.00 $P_\tau$=17.29</td>
<td></td>
</tr>
<tr>
<td>$T_\tau$=545.24 $P_\tau$=17.06</td>
<td></td>
</tr>
<tr>
<td>$T_\tau$=545.24 $P_\tau$=15.12</td>
<td></td>
</tr>
<tr>
<td>$T_\tau$=543.72 $P_\tau$=14.86</td>
<td></td>
</tr>
<tr>
<td>$T_\theta$ compressor (lb·ft)</td>
<td>-5.529</td>
</tr>
<tr>
<td>$T_\theta$ turbine (lb·ft)</td>
<td>14.86</td>
</tr>
</tbody>
</table>

Sta. 1: comp. inlet, Sta. 2: comp. exit, Sta. 3: turb. inlet, Sta. 4: turb. exit

Table 4. Torque calculation results (TR60 engine, $Ma = 0.5$, Sea level, International Standard Atmosphere).