$\frac{-\beta h_{fg}}{c_{pl}(T_A - T_B)} = \frac{5\sqrt{\pi}}{3}$ at low values of $\frac{\rho_l}{\rho_v}\left(\frac{T_\infty - T_B}{T_A - T_B}\right)$. An examination of equations (29), (30), and (31) reveals that

β is positive, if $T_s < T_\infty$
β is zero, if $T_s = T_\infty$

and

β is negative, if $T_s > T_\infty$.

Therefore the injection of helium gas into liquid oxygen results in the growth of gas bubbles and consequently cooling of the liquid oxygen is attained. Negative β may be obtained in case a steam bubble (although it is condensable) is injected into water.

Concluding Remarks

Approximate asymptotic solutions, including the influence of radial convection and diffusion, have been obtained for the growth and collapse of spherical bubbles in a nonisothermal system of varying composition. The results apply to growth and collapse controlled by heat and mass transport, which is characterized by uniform pressure throughout the system and by the movement of the phase boundary asymptotically proportional to the square root of time.

For bubble growth in boiling pure liquids and boiling binary mixtures, the results predicted by the source theory are in fair agreement with the exact solution of Scriven [1]. Other analyses or experimental results are not presently available to check the prediction for the bubble growth and collapse of the noncondensing, nonsoluble gas in liquids. This problem is currently being studied by the authors and their students. The accuracy of prediction may be improved by carrying more terms in equation (27) and by using more precise $T_s - x_s$ relationship than that expressed by equation (30).

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References

1 L. E. Scriven, "On the Dynamics of Phase Growth," *Chem. Eng. Sci.*, vol. 10, 1959, pp. 1–13.

2 H. K. Forster, "Diffusion in a Moving Medium With Time Dependent Boundaries," *Journal AIChE*, vols. 3–4, 1957–1958, p. 535.

3 H. K. Forster and N. Zuber, "Growth of a Vapor Bubble in a Superheated Liquid," *Journal Applied Physics*, vol. 25, 1954, p. 474.

4 M. S. Plesset and S. A. Zwick, "The Growth of a Vapor Bubbles in Superheated Liquids," *Journal of Applied Physics*, vol. 25, 1954, pp. 493–500.

5 H. S. Carslaw and J. C. Jaeger, "Conduction of Heat in Solids," second edition, Oxford Press, 1959.

6 C. Y. Han and P. Griffith, "The Mechanism of Heat Transfer in Nucleate Pool Boiling," Tech. Report No. 19, M.I.T., February, 1962.

DISCUSSION

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The authors treat an exciting problem, rendered the more fascinating by the heated arguments that usually develop between Mssrs. Plesset and Zwick on the one hand, H. K. Forster on the other, and N. Zuber on the third hand (see e.g. [7]).[4] In their paper, the authors perform a useful service in presenting some aspects of the Forster-Zuber method with greater detail and clarity than was available heretofore, as well as extending the method to binary mixtures and other cases. What this reader missed in the paper was a clear statement of (*a*) the problem that was being solved, and (*b*) the limitations of the method. (*a*) The problem, being time dependent, must have stated initial conditions in addition to the stated boundary conditions. The authors seem to restrict themselves to large-time behavior where the initial conditions do not matter. It should be so stated. This brings us to the second question, (*b*). Which features of the adopted mathematics prevent the use of the method at small times, and which features make it inexact at large times? In regard to the last question, is it assumption (7)?

It may be mentioned that the small-time behavior is all-important because the large-time behavior may never occur; the bubble's instability has been demonstrated by Mullins and Sekerka [8]. Another interesting feature of the small-time behavior is the very large pressure gradient which manifests itself in the surroundings of the growing nucleus [9].

For very large superheats the asymptotic formula

$$\beta = \varphi \frac{U}{1+\epsilon}, \quad U = \frac{c(T_\infty - T_s)}{h}, \quad 1 + \epsilon = \rho_v/\rho_e,$$

$$\varphi = \sqrt{\frac{3/2}{1-U}}$$

may be shown to be valid for the spherical model [10]. This expression (not limited to $0 < 1 + e << 1$) somewhat differs from those stated in Table 1.

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[4] Numbers in brackets designate Additional References at end of discussion.

Additional References

7 S. A. Zwick, "Growth and Collapse of Vapor Bubbles," GALCIT Hydrolab Rept 21-19, 1954.

8 W. W. Mullins and R. F. Sekerka, "Morphological Stability of a Particle Growing by Diffusion or Heat Flow," *Journal of Applied Physics*, vol. 34, 1963, p. 323.

9 G. Horvay, "Freezing Into an Undercooled Melt Accompanied by Density Change," *Proceedings 4th National Congress of Applied Mechanics*, ASME, 1962, p. 1315.

10 G. Horvay, "The Tension Field Created by a Spherical Nuclear Freezing Into a Less Dense Undercooled Melt," to be submitted shortly to a technical magazine.

Authors' Closure

The authors appreciate the contributions to this discussion by Dr. Horvay. As mentioned in the first paragraph of the Introduction and in the statement leading to equation (11), the results are restricted to the asymptotic stage of bubble development, where the growth has been initiated and is mainly governed by the transport of heat and mass from the surrounding liquid. The time intervals of the initial and intermediate stages which precede the asymptotic stage are very short for the types of bubble growth which we have studied (see references [1, 3, and 4]). The initial conditions were not mentioned, although they probably should have been, because the use of both Green's function and equation (11) implies the assumption of uniform initial temperature distribution. (See chapter 14 of reference [5].)

In regard to Dr. Horvay's question "*b*," we assume he means equation (7) by "assumption (7)." The mathematics employed in the analysis is, of course, necessary for the reduction of the problem as defined by equations (1), (2), and (3). The limitation on the use of the analysis to small times, however, is basically a question of physics since the governing differential equations would not then be the same.

Dr. Horvay's reference to small-time behavior is most significant and does constitute an important consideration. The systems at which we have directed our present attention involve physical processes in which the time corresponding to the initial and intermediate stages of bubble growth is a negligible fraction of the total time under consideration.