Aircraft Gas Turbine Rotating Compressor Disk Vibration

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This paper is concerned with the experimental investigation of vibration in aircraft jet-engine rotating compressor disks of two different configurations. The blades in the first configuration were small, with relatively low mass, and had an insignificant effect on the disk vibration except for centrifugal loading. The second configuration had relatively large blades which resulted in combined blade and disk modes. The test techniques, test results, and examples of some known phenomenon as they occur in lightweight aircraft jet-engine disks are discussed.


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Rotating disk and wheel vibration has caused much concern in the design of high-speed rotating machinery since before 1920. About this time several catastrophic failures of steam turbine wheels occurred in power-generating plants. Mr. Campbell (1) of General Electric Company directed a five-year investigation of turbine-wheel failure problems and was able to show that it was caused by wheel vibration. Mr. Campbell demonstrated the phenomenon of traveling waves in rotating disks. Since that time, considerable work has been done and a number of papers written on disk vibration. Most of these investigations have been on either steam-turbine wheels or laboratory specimens to determine the fundamental characteristics of rotating vibrating disks.

This paper covers more recent experimental work done on high-speed aircraft gas-turbine rotating disks including:

1. The use of today's test technique and instrumentation in studying disk vibration.
2. Examples of some known phenomenon as they occur in high-speed, light-weight, aircraft gas-turbine disks.
3. Sources of vibration excitation.

In the design of aircraft gas turbines there has been and is a continual requirement for lighter weight, longer life, and higher performance. Quite frequently these seem to be in odds, and only through the development of improved design and experimental methods has it been possible to accomplish them. The design of the disk has not escaped these demands, and extensive analytical and experimental work has been required to meet them. This experimental program was one phase of an effort to get a better understanding of aircraft jet-engine rotating-disk vibration. The experimental phase covers the vibration of two basic disk and blade designs as shown in Fig.1.

Configuration I, Fig.1(a), illustrates a design where the mass and stiffness of the blade is small as compared to that of the disk. They have little effect on the disk vibration except for the added centrifugal loading resulting from their mass. The Configuration II, Fig.1(b), design has much larger blades as compared to the disk. This results in combined blade and disk modes of vibration which do not exist for Configuration I.

ROTATING-DISK BASIC VIBRATORY BEHAVIOR

Campbell (1) showed that the flexural vibration of rotating disks has traveling waves which correspond to

\[ H = \frac{f_r}{r} + nN_s \]  
\[ M = \frac{f_r}{r} - nN_s \]

where

- \( H \) and \( M \) = frequency (cps) seen by a stationary observer for the forward and backward traveling waves respectively.
- \( f_r \) = natural frequency (cps) of the actual disk vibration (that indicated by a strain gage mounted on the disk).
- \( N_s \) = speed of disk rotation, rps
- \( n \) = number of waves or 1/2 the number of radial nodes.

Data from the rotating test were used in conjunction with these equations to determine the disk mode of vibration. The test data are then plotted on Campbell's frequency diagram as shown in Figs.10 through 14, 16 and 17.

The critical frequencies of a rotating disk increase with speed because of centrifugal loading according to the equation

\[ f_r^2 = f_o^2 + B N_s^2 \]

where

- \( f_r \) = natural frequency (cps) of the actual disk vibration (that indicated by a strain gage mounted on the disk).
- \( N_s \) = speed of disk rotation, rps
- \( f_o \) = natural frequency (cps) of the disk at zero speed for the respective mode.
- \( B \) = centrifugal force stiffening factor. It is a function of the disk configuration and is different for each mode.

Campbell (1) found that the failures in the steam-turbine wheel occurred at what is known as the disk critical speed. These occurred when the vibration characteristics of the disk and its speed satisfy the following equation

\[ f_r = nN_s \text{ or } (f_o^2 + B N_s^2)^{1/2} = nN_s \]  
\[ N_s = \left( \frac{f_o^2}{n^2 - B} \right)^{1/2} \]  

1 Underline numbers in parentheses designate References at the end of the paper.
On Campbell's frequency diagram this occurs when the backward traveling wave crosses the zero-frequency axis. At this condition any stationary disturbance around the disk's plane of rotation, such as unbalanced air flow through the stationary vanes, will excite vibration at the respective natural frequency. Except for disk critical speeds and at minor resonant speeds, vibration can only be excited by a periodic input with a frequency equivalent to that of the forward or backward traveling wave for the respective speed. (Random excitation is an exception, but this was not a part of the investigation.)

A minor resonant speed is the speed at which the backward traveling-wave frequency equals the frequency at which a fixed point on the disk passes stationary "per revolutions" F that may be present (for example, the struts in a frame). Minor resonance occurs when the speed equals

\[ N = \frac{f_r}{p + n} \text{ or } \frac{f_0}{((p + n)^2 - B)^{1/2}} \]  

**EXPERIMENTAL TEST APPARATUS AND PROCEDURE**

In conducting the experimental investigation on these disks, many of the techniques and much of the test setup used were common for both configurations. The items which were common are covered under this general heading to eliminate repetition. Only the test results and those things which are peculiar to a given configuration will be covered separately. The testing of a rotating disk was done in two steps. The first step was to determine the vibration characteristics of the disk statically, at zero speed. This, then was followed with the rotating test where the disk vibration was determined as a function of speed. The static information is important, since it is under this condition that the nodal pattern can be determined. In the rotating test the radial node can be determined with relative ease, but it is much more difficult to locate the circumferential nodes.

**Static Disk Vibration Test Setup**

As in all vibration tests it is important that the mounting of the disk for test dynamically simulates that in the engine. This is especially critical for umbrella and one-wave modes, since these modes have resulting forces which react beyond the immediate disk and its mating parts. The forces generated in the two- and above flexural modes are self-compensating because the summation of forces circumferentially around the disk at any point in time would equal zero.

In these tests the mounting of the disk was done using the adjacent parts as shown in Figs. 1(a) and 1(b) and then attached to a plate for mounting. Fig. 2 shows a photograph of the setup used for testing on the second configuration. The main question about the disk mounting was whether the difference in weight of the test vehicle as compared to the actual rotor would cause a significant change in the natural frequencies. The stiffness was duplicated by using actual hardware. In the static setup, testing was done with various weights attached to verify that the test vehicle was a reasonable simulation.

Vibration was excited in the disk with a variable-frequency electromagnet. The critical frequencies of the disk were determined by making frequency scans with the electromagnet at an approximately constant input and noting resonance as indicated by the disk response. These resonances occur when one of the disk's natural frequencies corresponds to the driving frequency of the magnet. The nodal patterns were determined using sand patterns as described by Kantorowicz (2), Figs. 5, 6 and 7. Or they were determined with a portable hand-held vibration pickup, and noting phase changes in the vibratory response around the disk with respect to the exciting force. The information obtained in this phase of testing is used in conjunction with the rotating test data to get a more complete picture of the disk vibration.

In rotating disk-vibration testing it is also important that the mounting of this disk for test represents the actual condition. This can be more
difficult to accomplish on the rotating test, because all parts are subjected to high centrifugal loading. The mounting of the disk affects both the magnitude of the steady-state stresses and the disk-vibration characteristics. On these two configurations the same basic mounting was used on the rotating test as on the static test. A check of the adequacy of the disk mounting on the drive was made by comparing the zero-speed disk critical frequencies for the respective mode with that determined in the static disk test.

The test setup and associated equipment is shown in Figs. 3 and 4 with the disk test assembly as shown in Fig. 1(a) assembled to the drive adapter. Prior to assembly the disk was instrumented with strain gages which is discussed later under instrumentation. The major pieces of mechanical equipment used were:

1. Drive which consisted of a 500-hp electric-motor magnetic clutch for speed control, speed-increasing gearbox, and an output spindle.
2. Test disk assembly.
3. Shroud (enclosure) for the outer portion of the disk and blades.
4. Siren (air chopper) for exciting vibration in the disk (an electromagnet did not provide sufficient excitation at high speed to excite vibration.)

The drive had speed capabilities for speeds up to 10,000 rpm with speed control through that range. The blades were enclosed in a shroud to reduce the power required to rotate the disk at
high speeds. This shroud also supported the air jets. These air jets were equally spaced around a circumference directed between the pitch and the tip of the blades. They were used to study the effect of blade vibration excited by a "per-revolution input on disk vibration.

The siren (air chopper) provided a pulsating air jet against the disk by having a slotted wheel driven with a variable-speed motor. This slotted wheel passes the exit to an air chamber, thus chopping the air at a frequency corresponding to the slot-passing frequency of the wheel. The excitation energy of the siren was set by controlling the air pressure. The location of the siren nozzle also affects the transmission of the energy from the siren to the disk. In cases where it is important to compare relative responses, the point of excitation and air pressure to the siren...
TABLE 1
INSTRUMENTATION FOR EXPERIMENTAL ANALYSIS OF ROTATING DISC VIBRATION

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Type and Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Angular Speed of Disc</td>
<td>60 per revolution sine wave generator to monitor speed of disc rotation.</td>
</tr>
<tr>
<td>2. Disc Displacement</td>
<td>Capacitance type pick up to determine disc vibratory frequency as seen by a stationary observer.</td>
</tr>
<tr>
<td>3. Disc Response</td>
<td>Strain gages to determine disc vibratory frequency, stress distribution and relative amplitudes.</td>
</tr>
<tr>
<td>4. Excitation Frequency</td>
<td>Siren output frequency (should be the same as that of No. 2 above).</td>
</tr>
<tr>
<td>5. Relative Excitation Energy</td>
<td>Inlet air pressure to siren.</td>
</tr>
</tbody>
</table>

were maintained. However, changing the siren nozzle locations gave a means of studying the transmittability of vibratory input at different locations.

INSTRUMENTATION

During test, instrumentation listed in Table 1 was used to determine the vibratory characteristics and stress distribution of the disk during rotation.

The accuracy of speed and frequency measurements are critical in order to determine the vibratory modes accurately as can be seen by examining equations (1) and (2). Speed was measured using a 60-per-revolution sine-wave generator attached to the output shaft of the drive and counting the pulses on an electronic counter. Frequencies were determined by beating out the Lissajous patterns on oscilloscopes with an oscillator and measuring the oscillator frequency on the electronic counter.

The displacement pickup was used only to determine the vibratory frequency seen by a stationary observer. It was not calibrated for amplitude of displacement, since the strain gages gave the stress level which was of most interest. The excitation and the displacement pickup frequencies should be the same. The displacement pickups were used to eliminate the possibility of being misled by the disk responding at harmonics of the excitation frequency.

![Fig. 7 Nodal patterns. Note additional circumferential mode for Configuration I](image-url)

Electrical resistant-type strain gages were installed on the disk to determine amplitude of stress and modes of vibration. The leads were brought out through a slip ring. The strain gages instrumentation was done so that both cen-
trifugal and vibratory strains could be determined. To compensate for changes in lead and slip-ring contact resistance with temperature and speed, a three wire lead-out was used for the steady-state strain measurement. Only a two wire lead-out is required for the dynamic strain measurement. Care must be used in selecting a type of strain gage that matches the material for the steady-state strain measurement, because the temperature of the disk increases with speed thus giving a false indication of strain, if the incorrect type of gage is used.

Although centrifugal stresses were measured and are an important part of high-speed disk designs, this paper is concerned only with the disk dynamic behavior. Since the stress fields in a disk are bi-axial, both radial and tangential strain gages were required at each location to determine the actual stress, reference (3).

The strain gages were also installed back to back, on both sides of the disks, at the same locations so that the membrane and bending stresses could be separated.

In conducting the test, data were obtained:
1. Operating the disk, at constant speed and making frequency scans with the siren and,
2. Making speed scans using air jets on the blade.

In the first method after selecting the rotational speed point at which to obtain data the next step was to determine the disk's natural frequencies by making plots, Fig.8, of the disk response as indicated by the strain gage versus excitation frequency. Following these scans, frequencies and strain measurements were recorded at each of the disk's natural frequencies. Data were obtained at 4 to 6 speed points throughout the operating range to determine the effect of speed on the disk vibration characteristics. Testing was done with the nozzle of the siren di-
rected at different locations on the disk to check transmittability. For example, on Configuration I in Fig.1, data were obtained with the siren directed at the bearing location on the stub shaft to determine if an input from the bearing could excite vibration in the disk.

In the second method, speed scans were made using different "per revolution excitation. The "per-revolution excitation was done using the air jets in the shroud which were directed on the blades. Data were recorded at the disk and blade natural frequencies the same as during the constant-speed, variable-frequency excitation testing.

TEST RESULTS

Configuration I

On this configuration the nodal patterns for the static conditions were determined using sand patterns as shown in Figs.5, 6 and 7. They are, in general, axial symmetrical with the higher modes being somewhat distorted. Note the modes, Fig.7, with an additional circumferential node. In the rotating test a second set of modes were also found which corresponds to these modes of vibration.

Fig.8 shows disk response plotted as a function of excitation frequency for the 6000-rpm speed point. This plot is typical of the plot made at each speed point. Each of the peaks in this curve indicate a condition of resonance where frequency of either the forward or backward traveling wave corresponds to the excitation frequency. Fig.9 shows the same information as that encircled in Fig.8 with an expanded frequency scale. Also, indicated on this plot is the identity of the disk vibratory mode which corresponds to each of these peaks. Some of the peaks are made up of more than one mode. This condition occurs when the frequencies of the backward or forward traveling wave of two modes are equal. In these cases, the frequency of the disk as indicated by the strain gage is made up of two or more distinct frequencies.

Fig.9 also shows that in some cases for a
given mode there are two response peaks separated only by a few cycles per second. This phenomenon is one that has been previously observed on dynamically imperfect disks. Tobias and Arnold (4) and (5) discuss this phenomenon and show that the amplitude of the disk response reduces with an increased degree of the imperfection. The amount of imperfection in this disk was very small. According to the design the disk was nominally axial symmetrical and dynamically perfect. Any deviation from this would be only that allowed in the machining tolerance of a precision.

aerospace jet-engine part. The data show by the two peaks for the same mode that the disk was not dynamically perfect. This illustrates how difficult it is to produce a dynamically perfect disk and points out a problem that exists in trying to evaluate damping of rotating disks based on the relative response. The part-to-part variation in response, because of undefined dynamical imperfections can be relatively large. The specific modes of vibration were determined from the frequency of disk vibration (strain-gage output), excitation frequency (frequency seen by a stationary observer), the speed of disk rotation, and equation (1) or (2).

Campbell's frequency diagram is a plot of the solutions to these equations and is a visual aid in analyzing rotating disk vibration. Fig.10 is a sample diagram with legend defining its parts including per-revolution excitation from ball and roller bearings. The test data were plotted on the frequency diagram as shown in Figs.11 through 14. These figures were selected to illustrate the vibration characteristics of this type of rotating disk.

These frequency diagrams also show that the natural frequency of the disk increases at such a rate with speed that the backward traveling wave never crosses the zero-frequency line. Thus, this configuration does not have a disk critical speed. Table 2 gives a listing of all vibratory modes found and Fig.15 shows a plot of the natural frequency of each mode as a function of the number.

Fig. 12 Frequency diagram at the two-wave mode for Configuration I

Fig. 13 Frequency diagram of the four-wave mode for the Configuration I

Fig. 14 Frequency diagram of the seven-wave mode for Configuration I
Fig. 15 Natural frequencies of the vibratory modes versus number of waves for Configuration I

Fig. 16 Frequency diagram of the umbrella and low-frequency two-wave mode for Configuration II

The ball or roller-passing frequency of an antifriction bearing can be predicted very accurately based on the geometry of the bearing and the shaft speed as described by Palmgren (6). The amplitude of this excitation from a normal bearing is very low. A spall in one of the races, foreign material in the bearing or a chip in one of the balls can result in a significant increase in the exciting force. The ball-passing frequency is different for a fixed point on the inner and outer race, because of the rotation of the shaft. The sum of these ball or roller-per-revolution excitations is equal to the number of ball or rollers in the bearing.

Conditions of resonance occur with respect to the bearing when

1. The natural frequency of one of the disk modes corresponds to the ball-passing frequency with respect to the inner bearing race and,

2. When the frequency of either the forward or backward traveling waves corresponds to the ball-passing frequency with respect to the outer bearing race.

This is obvious in that a flaw on the inner bearing would rotate with the disk. The excitation frequency of an input rotating with the disk has to be equal to the disk's natural frequency for resonance to occur. A flaw in the outer race would remain stationary and would appear as a stationary observer. Thus, resonance would occur when the ball-passing frequency is equal to the frequency observed by a stationary pickup (forward or backward traveling wave). On the frequency diagram a condition of resonance exists at the speed where the bearing inner or outer race "per-revolution line crosses the respective frequency line as shown in the frequency diagram, Figs.11 through 14.

Configuration II

The web of the disk, portion between the bore and the disk rim, followed the same general behavior as for Configuration I with similar results being obtained. However, Configuration II also had combined blade and disk modes. These combined modes were more difficult to excite and evaluate in both the static test and in the rotating test.

In the static test it was necessary to wedge the blades in the dovetail slots to simulate the locked up condition that occurs with centrifugal loading. Also, concerning frequency some of the modes were located very close together as shown in Fig.18. This figure is for 6000 rpm but a similar condition existed at zero speed.
When the frequency of two modes coincided, the resulting nodal patterns are complex and difficult to interpolate. Fourier's series analysis can be used by solving for the wave form of the deflection around the circumference of the blade, but because of the asymmetry of the individual modes, this does not completely solve the problem. In the static testing an electromagnet was used to excite vibration and the nodal patterns were traced out with a portable hand-held pickup. The setup for this is shown in Fig. 2.

The test procedure followed in the rotating test was basically the same as described earlier. The method of exciting vibrations had to be changed, because the transmittability of energy from the web of the disk to the rim and blade was so low that blade vibration could not be excited by directing the siren nozzle on the web. This design had disk critical speeds. Vibratory response occurred when the test speed equalled one of the disk's critical speeds. The only method found to excite these combined blade and disk modes at all speeds was to connect the siren output to a number of the air jet nozzles directed on the blades and exciting at the backward traveling-wave frequency. Even with this method, vibration could only be excited at very low amplitudes. It is felt that the problem with excitation was mainly from dynamical imperfections which would be normal due to blade-to-blade variation and aero-dynamic damping.

Test data were obtained on the umbrella, one, two, and three-wave modes. Figs. 16 and 17 are sample frequency diagrams for these modes. These diagrams show that this disk with its blades had disk critical speeds in the speed range of the investigation for the lower frequency to wave and both of the three-wave modes. These critical speeds occurred at 3800, 2000, and 6600 rpm, respectively and are the speeds at which vibrations would be excited by stationary disturbance around the blades path of rotation. The disturbances could result from inlet distortion, struts in the air-flow annulus, out-of-roundness of the compressor casing or by an off-schedule vane.

The foregoing disk critical speeds were also observed in making speed scans. In these scans a relative high response occurred over a wide speed range. (In general, when exciting a rotating disk with the siren the resonant points were characterized by a sharp peak similar to those shown in Figs. 8 and 9.) The speed range over which this response occurred was so wide that it was not possible to define the natural frequency of the disk by the response peak. Tobias and Arnold (4) also made this same observation and investigated the response of a stationary in space mode as a function of speed and amplitude of excitation. They showed that both the amplitude of response and speed range over which the response occurred increased with an increase in amplitude of excitation.

A condition to be avoided as discussed by Dabagyan (7) is one in which the blades' natural
frequency is close to that of the disk at the
disk critical speed or any other condition of
resonance. In this case if the blades' natural
frequencies are distributed so that only one of
the blades' natural frequencies corresponds to
the blade and disk natural frequency, the blade
will act as an auxiliary mass damper. Its ampli-
tude can be extremely high when vibration is ex-
cited in the disk. If all blades were tuned to
this frequency, they all would take out energy so
the maximum amplitude would be significantly re-
duced.

Fig. 18 shows a plot of the natural frequency
of the modes of vibration versus number of waves,
along with the natural frequency of the blades.
This particular figure is for the 6000-rpm point.
It gives a visual picture of how the blade crit-
tical frequency compares with that of the blade
and disk modes of vibration.

The blade first flexural mode is relatively
close to the lower frequency family of the com-
bined blade and disk modes. This condition should
be avoided if the disk critical speeds for the
respective mode occurs in the engine operating
speed range. (In this particular case the disk
critical speed was below the operating range.)

The blades' second flexural mode is signifi-
cantly above any of the combined blade and disk
modes.

The combined blade and disk modes were highly
damped and could only be excited to low levels.
The damping in Configuration II is significantly
higher than that of Configuration I based on the
amplitude frequency ratio as given by Harris and
Crede (8).

\[
Q = \frac{W_n}{\Delta W} = \frac{n}{\Delta}
\]

\[
\Delta = \frac{\Delta W}{W_n}
\]

where

- \( W_n \) = natural frequency
- \( \Delta W \) = difference in frequency of the two sides
  of the response curve at the half-power
  point (0.707 of peak response)
- \( \Delta \) = logarithmic decrements
- \( Q \) = response factor

The logarithmic decrements determined for the
two designs varied from 0.008 to 0.022 and 0.021
to 0.050 for Configuration I and II, respectively.

CONCLUSIONS

In this program the vibration characteristics
of aircraft jet-engine rotating disks were studied
using the latest instrumentation and test tech-
niques. Use of an air siren as a vibration ex-
citer was incorporated to provide adequate energy
to excite vibration at high rotating speeds where
aerodynamic damping was high.

The test results showed that:

1. The small manufacturing variations that
   occur in a precision aircraft jet-engine disk con-
   figuration are sufficient to make the disk dy-
   namically imperfect so that the amplitude of re-
   sponse of one disk is not a good measure of damp-
   ing for all disks of that configuration.

2. The per-revolution output of a defective
   antifriction bearing can be resonant with disk
   natural frequency and excite vibration.

3. On designs such as shown in Configuration
   II, the blades and disk combine to form a multi-
   degree of freedom vibration system which has more
   than one mode for a given number of waves.

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