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A STUDY OF CALCULATIONAL MODEL FOR PREDICTING SURGE LINE IN A CENTRIFUGAL COMPRESSION SYSTEM WITH COOLER AND CONDENSER

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ABSTRACT

A calculational model for predicting the surge line of a turbocompressor system has been established which is composed of a centrifugal compressor, duct,coolers and condensers (gas-liquid separator). The influences of various parts of the system on the surge point have been studied in detail, and some discussion has been focused on the coolers and condensers. The influences of cooler ,condenser and thermodynamic parameters of working medium on stability of the system is calculated and explained, This analysis is useful for predicting system character during the of design compressors and systems, as well as safety for operation of practical industry process.

NOMENCLATURE

A	Area of Duct or Flow Passage
Cp	Specific Heat in Constant Pressure
Cv	Specific Heat in Constant Volume
h	Enthalpy
k	Adiabatic Exponent
L	Length of Duct or Flow Passage
m	Mass Flow rate
M	Mass in a Control Volume
P	Pressure
Q	Heat Quantity
R	Gas Constant
t	Time
T	Temperature
V	Volume
W	Work Quantity
ρ	Density

Subscripts

c	Compressor
cv	Control Valve
E	Exit
i	Inlet of Compressor
L	Liquid
p	Plenum or Cooler
s	Condenser
T	Valve

INTRODUCTION

It is well known that the characteristic performance of a turbo-compressor system at the range near the peak of the pressure-rise curve is limited by aerodynamic instabilities. Rotating stall and sur are the main form of the instabilities. Generall rotating stall is preceded by separation flow in the diffuser or in the impeller. Surge is a system

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oscillation associated with reversed flow that causes very strong hydrodynamic impacts to parts of the compressor and system. Predicting and avoiding system instabilities has been the focus of many researchers and industry. For axial-flow compression systems, a model for stall and surge has been established by Moore (1984), Moore and Greitzer(1986). Many researchers have studied how to improve the instability character of the system or increase the stable operation range of compressors, for example, Adamczyk (1991), Day(1991), Arnulfi and Massardo(1996). However, research for centrifugal compressor systems are comparatively few. For centrifugal compression systems, surge is more dangerous than stall, because it may damage the system. In order to solve the problem Dai and Gu (1995) established a model to predict the temperature rise during deep surge in a centrifugal compression system and obtained a good result which agreed in tendency between the predictions and experiments. Dai and Gu (1996), then, proposed a model which describes the surge transients in centrifugal compression systems with a vaned diffuser. The model is established on basis of the pressure-rise equations of each component of the compressor. However, In fact, the practical compression system does not only include the compressor and diffuser,but ducts, coolers and a condenser. The main idea of the present work is to make a model predicting the surge point of a complex system which is composed of a centrifugal compressor, diffuser, duct, cooler and condenser. The model will be useful to help the designers predict the instability character of the system during design , improve it, and understand various influences of each component of the system on instability. For convenience , a lumped parameter method is applied to solve the practical engineering problem.

MODELING AND GOVERNING EQUATIONS

The centrifugal compressor system studied is shown in Fig. 1

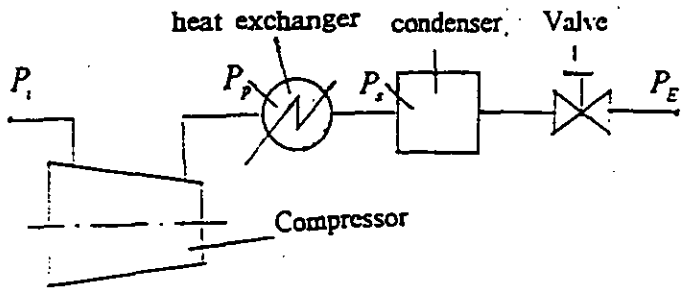

Fig. 1 Centrifugal Compression System

We assume that the flow in the pipe is incompressible and the flow in the plenum is compressible. The compressor is explained by the curve of pressure-rise vs flow rate. The working medium is ideal gas.

First, the pressure equations of each system component are established, then, linearized by small perturbation method. Then, the eigenvalue of the equations are solved which is used to determine the stability of the system. The equations are as follows:

COMPRESSOR

The gas pressure at the inlet of the compressor is P_i, The mass flow rate passed in it is m_c. The momentum equation of this component is given by :

$$P_i + \Delta P_C - P_P = \frac{L_c}{A_i}\frac{dm_c}{dt} \qquad (1)$$

where Δp_c is steady pressure rise performance curve of the compressor, L_c is equivalue length of the flow passage of the compressor , A_i is area of cross section of the inlet duct of the compressor, Eq.(1) is analyzed by small perturbation method and becomes

$$\frac{d\delta m_c}{dt} = \frac{A_i}{L_c}\left(\delta P_i + \frac{\partial\overline{\Delta P_c}}{\partial m_c}\delta m_c - \delta P_p\right) \qquad (2)$$

PLENUM AND COOLER

The volume of the cooler is assumed so great that the internal pressure is uniform and denoted by

P_p. The gas flow rate of the cooler is denoted by m_p and its temperature by T_p. According to the theory of a varying mass flow system, the energy equation of the control volume can be written as:

$$Q = \frac{dU}{dt} + \sum hm + W_{cv} \tag{3}$$

Neglecting change of kinetic energy and potential energy in the control volume,i.e. $W_{cv} = 0$, Eq.(3) can be written as

$$\frac{d\left(Mc_v T_p\right)}{dt} = Q + m_c c_p T_c - m_p c_p T_p \tag{4}$$

If the mass of gas in the plenum/cooler is M, The mass conservation equation in the plenum is given by

$$\frac{dM}{dt} = m_c - m_p \tag{5}$$

Assuming the gas volume of the cooler to be V_p and the volume of liquid is so little that it can be neglected, substituting the state equation $P_p V_p = MRT_p$ into Eq.(4) and analyzing by small perturbation analysis, we get

$$\frac{c_v}{R} V_p \frac{d\left(\delta P_p\right)}{dt} = \delta Q + c_p m_c \delta T_c + c_p T_c \delta m_c - c_p m_p \delta T_p - c_p T_p \delta m_p \tag{6}$$

Let $\delta Q' = \delta Q + c_p m_c \delta T_c - c_p m_p \delta T_p$ and Assume instability heat transfer is the same phase as the pressure wave as proposed by Greitzer(1981). Therefore, $\delta Q' = \mu_Q \delta P_p$, where μ_Q is heat pressure influence coefficient and can be taken by a constant . A temperature- pressure coefficient is defined as $\mu_T = \dfrac{\delta T_p}{\delta P_p}$, Then ,the small perturbance form of Eq.(5) is

$$\frac{d\delta M}{dt} = \delta m_c - \delta m_p$$

and

$$\delta m_p = \delta m_c - \frac{d\delta M}{dt}$$

Substituting the equation above into Eq.(6)

$$\begin{aligned}
\frac{c_v}{R} V_p \frac{d\left(\delta P_p\right)}{dt} &= \delta Q' + c_p T_c \delta m_c - c_p T_p \delta m_c + c_p T_p \frac{d\left(\delta M\right)}{dt} \\
&= \delta Q' + c_p\left(T_c - T_p\right)\delta m_c + c_p \frac{d\left(\delta M T_p\right)}{dt} - c_p M \frac{d(\delta T_p)}{dt} \\
&= \delta Q' + c_p\left(T_c - T_p\right)\delta m_c + c_p \frac{V_p}{R} \frac{d\left(\delta P_p\right)}{dt} - c_p \frac{P_p V_p}{R T_p} \mu_T \frac{d(\delta P_p)}{dt}
\end{aligned}$$

this is

$$\frac{V_p}{R} C_p \left(\frac{1}{k} - 1 + \frac{P_p}{T_p}\mu_T\right) \frac{d\left(\delta P_p\right)}{dt} = \mu_Q \delta P_p + C_p\left(T_c - T_p\right)\delta m_c$$

Then we get the final equation as following

$$\frac{d\left(\delta P_p\right)}{dt} = \frac{\mu_Q}{\left[C_1\right] c_p} \delta P_p + \frac{T_c - T_p}{\left[C_1\right]} \delta m_c \tag{7}$$

where $\left[C_1\right] = \dfrac{V_p}{R}\left(\dfrac{1}{k} - 1 + \dfrac{P_p}{T_p}\mu_T\right)$.

CONDENSER

The gas in the condenser will be in the saturation state When its temperature decreases below saturation temperature under a pressure, the liquid is separated from working medium and exhausted by a by-pass pipe, which keeps the gas volume in the condenser constant . Using the energy equation of varying-mass flow system Eq.(3) we have

$$\frac{dU}{dt} = m_p h_p - \left(m_s h_s + \Delta m h_L\right) \tag{8}$$

where h_p and h_s are enthalpy of the gas at the inlet and outlet of the condenser respectively. h_L is enthalpy of medium in liquid. m_E is the gas mass flow rate after the condenser. The liquid mass condensed from working medium is $\Delta m = m_p - m_E$ and

$$\frac{dU}{dt} = \frac{d(Mc_v T_s)}{dt} = \frac{V_s c_v}{R}\frac{dP_s}{dt}$$

where V_s is gas volume of the condenser. Substituting them into the Eq.(8), we get

$$\frac{V_s c_v}{R}\frac{dP_s}{dt} = (c_p T_p - h_L)m_p - (c_p T_s - h_L)m_E \tag{9}$$

Analyzing Eq.(9) by small perturbation and suggesting $\delta h_L = 0$, we get

$$\frac{V_s c_v}{R}\frac{d\delta P_s}{dt} = c_p m_p \delta T_p - c_p m_E \delta T_s + (c_p T_p - h_L)\delta m_p - (c_p T_s - h_L)\delta m_E \tag{10}$$

It is assumed that condensed process in condenser is always kept in a two-phase situation so we use a state equation

$$\frac{dP_s}{dT_s} = \frac{r}{(v_s - v_L)T_s} \tag{11}$$

where r is latent heat of evaporation, v_s is specific volume of gas state, v_L is specific volume of liquid state. The Eq.(11) can approximately be written as

$$\delta P_s = \frac{r}{(v_s - v_L)T_s}\delta T_s \tag{12}$$

Variables such as T_S, P_S, m_E etc. are obtained after calculation of condenser parameter in stability state.

Substituting $v_L = 1/\rho_L$ *and* $v_s = RT_s/P_s$ into Eq.(12), we get

$$\delta T_s = \left(\frac{RT_s}{P_s} - \frac{1}{\rho_L}\right)\frac{T_s \delta P_s}{r} \tag{13}$$

The Eq.(13) is used to take place of the δT_S in Eq.(10).

Let

$$[C_2] = \left(\frac{RT_S}{P_S} - \frac{1}{\rho_L}\right)\frac{T_S}{r}$$

and

$$\mu_T = \delta T_P / \delta P_p$$

we have

$$\frac{V_s c_v}{R}\frac{d\delta P_s}{dt} = c_p m_p \mu_T \delta T_p - c_p m_E [C_2]\delta P_s + (c_p T_p - h_L)\delta m_p - (c_p T_s - h_L)\delta m_E \tag{14}$$

Now, Eq.(7) is used to change the perturbation formation equation of the Eq.(5), and suggesting $[C_3] = \frac{V_P}{RT_P}\left(1 - \frac{P_p}{T_p}\right)\mu_T$,after prolix deducing we get

$$\delta m_p = \left[1 - \frac{[C_3](T_c - T_p)}{[C_1]}\right]\delta m_c - \frac{[C_3]\mu_Q}{[C_1]c_p}\delta P_p \tag{15}$$

Substituting Eq.(15) into Eq.(14) and reducing, we get final equation on condenser:

$$\frac{d\delta P_s}{dt} = \frac{R}{c_v V_s}(c_p T_p - h_L)\left[\frac{[C_3](T_c - T_p)}{[C_1]}\right]\delta m_c + \frac{R}{c_v v_s}\left[c_p m_p \mu_T - (c_p T_p - h_L)\frac{[C_3]\mu_Q}{[C_1]c_p}\right]\delta P_p - \frac{kR}{v_s}m_E[C_2]\delta P_s - \frac{R}{c_v v_s}(c_p T_s - h_L)\delta m_E \tag{16}$$

EQUATIONS OF SYSTEM AND CALCULATION.

The small perturbation equations of the compressor , cooler and condenser are described respectively by Eq.(2),Eq.(7) and Eq.(16). These equation are simultaneously solved to get closure for δm_c , δP_s , δP_p . The closed equations can be solved to determine the small perturbation character of flow rate and pressure in whole compression system.

The equations are rewritten as:

$$\begin{cases} \dfrac{d\delta m_c}{dt} = a_1\delta P_i + b_1\delta m_c + c_1\delta P_p & (2) \\ \dfrac{d\delta P_p}{dt} = b_2\delta m_c + c_2\delta P_p & (7) \\ \dfrac{d\delta P_s}{dt} = b_3\delta m_c + c_3\delta P_p + d_3\delta P_s & \\ \qquad + e_3\delta m_E & (16) \end{cases} \quad (17)$$

where P_i , inlet pressure of the system, can be assumed as a constant.Therefore, $\delta P_i = 0$,At the same time ,the outlet pressure of the system is considered constant i.e. $P_E =$ const. According to the character of the valve, the pressure drop of valve is of parabolic law, so we can get

$$P_s - P_E = \frac{K_T}{2}\frac{m_E^2}{\rho_T A_T^2} \quad (18)$$

where K_T is flow coefficient, A_T is the area of flow cross-section while the valve is closed in some degree. It is assumed that $\rho_T =$ constant and $\delta P_E = 0$ and $\delta A_T = 0$, if the close-open velocity of the valve is slow.Analyzing Eq.(18) by small perturbation , we have

$$\delta P_s = \frac{K_T m_E}{\rho_T A_T^2}\delta m_E \quad (19)$$

Substituting the Eq.(19) into the third equation of Eqs.(17), we get

$$\frac{d\delta P_s}{dt} = b_3\delta m_c + c_3\delta P_p + \left(d_3 + \frac{\rho_T A_T^2}{K_T m_E}e_3\right)\delta P_s \quad (20)$$

where m_E is obtained by calculation of Eq.(18) in stability state ,so the boundary condition at inlet and outlet is induced to Eqs. (17).

Because ordinary differential equations produced by small perturbation ,Eq.(17), have zero solution, it is necessary to solve the eigenvalue of matrix for system stability analysis. If the real part of eigenvalue is greater than zero, the system is stable at this flow rate. On the contrary, if the system is unstable. The model can be used to judge whether surge occurs in the system or not.

CALCULATION EXAMPLE AND ANALYSIS

Experiments from a small-sized surge experimental facility for centrifugal compressors, provided by Gysling(1991), was chosen for sample calculations. The calculational results of the system which includes the plenum was in good agreement with the data of the experiment. In the present work, we focus on the character of the surge point of the system with the condenser .

INFLUENCE OF CONDENSER ON SURGE POINT

The influence of the condenser on the surge point of the compressor are shown in Fig.2. In the figure, the solid lines show the surge points of the centrifugal compressor system with the condenser, dashed lines show the surge line of the system without condenser but with two different volumes.The turbocompression system with the condenser is similar to a system with a very large plenum, in which there is a higher flow rate at surge. The stable operating range of the system becomes narrow,and it is more obvious in the case of low rotation speed. In addition, the calculations show that many parameters do not influence the surge point of the system because of the action of the condenser.

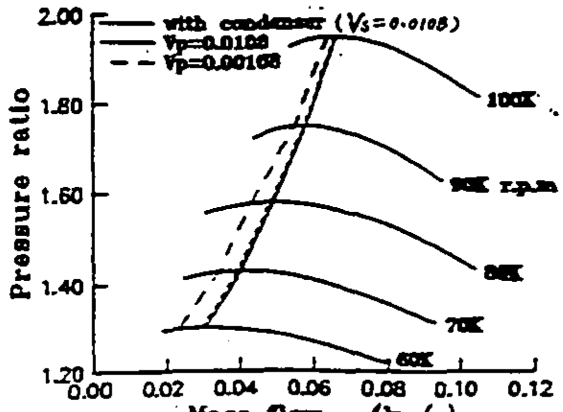

Fig.2 Influence of condensation on surge flow rate

INFLUENCE OF OTHER PARAMETER OF THE SYSTEM ON STABILITY

For the complex compression system with condenser, the influence of pressure coefficient of temperature, μ_T , and heat pressure affect coefficient, μ_Q , on system stability are discussed and shown in figure 3 and figure 4. In figure 3, there is a critical line of μ_T versus rotational speed. While μ_T is less than the critical value ,the system is unstable. The influence of μ_Q on surge is shown in Fig 4. the value of μ_Q influences on flow rate of surge point. The larger absolute value of μ_Q and lower rotational speed of compressor, the more variation of flow rate of surge point. This means surge point moves to left or right and influences the operation range of the system. In Fig. 4, m_{so} is maximum flow rate at peak point of pressure rise performance curve, m_s is flow rate of the sample case under a μ_Q and rotational speed. $\delta m_s = m_s - m_{so}$. Based on the analyses and calculations we found that the stability of a centrifugal compression system with condenser is very complex and difficult to accurately predict because the relations between various parameters and their influences on each other are delicate. In the present work the calculations can only be used to explain variation tendency of influences of various parameters on surge line in approximate quantity.

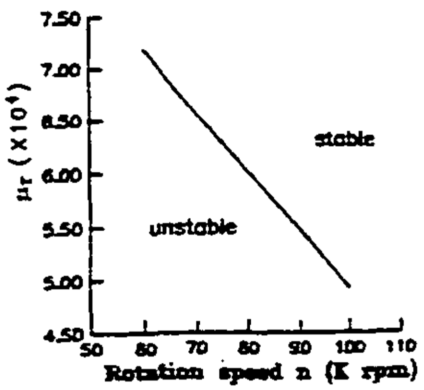

Fig.3 Influence of μ_T on stability

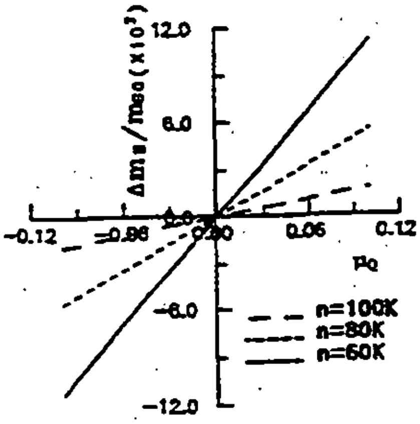

Fig.4 Influence of μ_Q on surge flow rate

CONCLUSION

The stability of a complex compression system with a cooler and a condenser was studied by the lumped parameter method and small perturbation theory in the paper. Some conclusions are obtained by the prime numerical analysis:

(1) The system with condenser is more unstable than that without condenser. Its surge flow is larger and operation range narrower. The surge flow of the system is not able to be decreased by adjusting constructure parameters of the system

(2) Critical value of μ_T exists at a special rotational speed. If it is larger than that critical value , the system is always unstable at any flow rate in the left branch of performance curve .

(3)Influences of μ_Q at different revolution speed are different on surge flow rate of system, which is useful for design cooler and condenser in a varying speed system.

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REFERENCE

Adamczyk, J. J., Celestina, M. L. and Greitzer, E. M., 1991, The Role of Tip Clearance in High-Speed Fan Stall, ASME, 91-GT-83.

Arnulfi,G. L., and Massardo,A. F., 1996, Comparison Between Complete Hilbert Transform and Simplified Solutions of The Moore Rotation Stall Model, ASME , Paper 96-GT-140.

Dai, Ji, Gu, Chuangang and Miao, Yongmiao, 1995, Temperature Rise During Deep Surge in Centrifugal Compression system, ASME, paper 95-GT-438.

Dai, Ji, Gu, Chuangang and Miao, Yongmiao, 1996,Surge Transients in Centrifugal Compression System with Vaned Diffuser, Proc. of the 2nd International Conference on Pump and Fan, Vol. 1, pp. 365-375.

Day,I.J., 1991, Active Suppression of Rotating Stall and Surge in Axial Compressors, ASME, Paper 91-GT-87.

Greitzer, E. M.,1981,The Stability of Pump Systems the1980 Freeman Scholar Lectrue,ASME, J.of Fluids Eng., Vol. 103, pp.193-242.

Gysling, D. L., Dugundji, J., Greitzer, E. M. and Epstein, A. H., 1991, Dynamic Control of Centrifugal Compressor Surge Using Tailored Structures,ASME, J. of Turbomachinery, Vol.113, pp. 710-722.

Moore,F.K.,1984,A Theory of Rotating Stall of Multistage Compressor ,ASME,J.of Eng of Gas Turbin and Power,Vol 106,pp.313-336.

Moore, F. K. and Greitzer, E. M.,1984,A Theory of Post-Stall Transients in Axial Compressors System, ASME, J. of Eng. for Gas Turbine and Power, Vol. 108, pp. 68-76.

Moore, F. K.,1991,Theory of Finite Disturbance in a Centrifugal Compression System with a Vaneless Diffuser, ASME Paper 91-GT-82.

Ubaldi, M., Zunino, P. and Barigozzi, G., 1996, An Experimental Investigation of Stator Induced Unsteadiness on Centrifugal Impeller Outflow, ASME, J. of Turbomachinery, Vol. 118, pp. 41-51.

Wu,peiyi and Ma, yong,1993, Thermodynamics of variable mass system and its Application, (in Chinese), Publishing House of High Education.