NUMERICAL ANALYSIS OF 3-D UNSTEADY HOT STREAK MIGRATION AND SHOCK INTERACTION IN A TURBINE STAGE

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ABSTRACT
A 3-D unsteady flow computation has been performed for a transonic first turbine stage under the influence of streaks of hot gas exiting the combustion chamber. Realistic flow conditions are obtained by using a non-equal stator-to-rotor pitch, a single-streak/multi-stator channels configuration and periodic boundary conditions. The resulting unsteady shock waves system and the hot streaks migration as well as the shock wave/streak interaction are presented and discussed. In addition, the time-average of the periodic unsteady solution is analyzed and compared with a steady-state computation. The steady-state solution matches the time-averaged one in terms of the pressure field and the maximum stagnation temperature on the rotor blade surface. However, the rotor blade temperature patterns are different with a stronger radial secondary flow present in the time-averaged solution due to the retention of the circumferential streak variations at the stator/rotor interface.

NOMENCLATURE

\[ \begin{align*}
  c & : \text{speed of sound} \\
  h_i & : \text{stagnation enthalpy} \\
  p & : \text{static pressure} \\
  p_t & : \text{stagnation pressure} \\
  R & : \text{radius} \ (R = \sqrt{y^2 + z^2}) \\
  t & : \text{time (non-dimensionalized with period)} \\
  T_t & : \text{stagnation temperature} \\
  u_x, u_y, u_R & : \text{Cylindrical velocity components in} \ (x, \theta, R) \\
  W & : \text{absolute velocity (stator)} \\
  \gamma & : \text{ratio of specific heats} \\
  \rho & : \text{static density} \\
  \phi & : \text{characteristic variable} \\
  \Omega & : \text{angular speed} \\
  \omega & : \text{vorticity} \\
  (\cdot)_{\text{inlet}} & : \text{inlet undisturbed flow (no streak)} \\
  (\cdot)_{\text{rotor}} & : \text{rotor, stator}
\end{align*} \]

1 INTRODUCTION
The accurate prediction of the heat loads encountered by the first turbine stage of a gas turbine is of primary importance for the design of an efficient cooling system. By construction, a modern first turbine stage has to operate under circumferential and radial non-uniform inflow temperature conditions produced by the combustor, as well as transonic flow conditions. This, and the inherent unsteadiness of a stator/rotor configuration, creates a highly complex three-dimensional environment for the turbine designer.

In this paper a three-dimensional, inviscid, periodic unsteady solution is presented for the flow in a generic industrial transonic axial turbine stage. This flow includes non-uniform inflow conditions produced by streaks of hot gas discharged from the combustion chamber. A realistic configuration comprising a stator-to-rotor pitch ratio of 5:3 and a streak count different from the stator blade count has been chosen. This is important as the time-averaged rotor blade surface temperature is directly dependent on the streak count and residence time in the passage. The problem of temperature redistribution in an axial flow turbine stage has been analyzed by several authors, see for instance Butler et al. (1986), Krouthen and Giles (1988), Ni and Sharma (1990), Harasgama (1990), Dorney et al. (1990), and Takahashi and Ni (1990). In particular, the numerical studies of Ni and Sharma, and Dorney et al. tend to reproduce the migration of one midspan, circular hot streak of fluid, experimentally investigated by Butler et al. Notice that in contrast to the present analysis, the above mentioned studies report on a turbine operating under subsonic flow conditions, where the level of unsteadiness is much lower than for the transonic case. It has been experimentally observed that the periodic unsteady interaction between a shock wave and a turbine airfoil can cause considerable effects in terms of blade loading and heat transfer, see for instance Doorly and Oldfield (1988), Johnson et al. (1988), and Collie et al. (1992). As they occur simultaneously in a transonic first turbine stage and as they both are primarily
driven by inviscid mechanisms, it appears quite natural to simulate streak migration and unsteady shock wave interaction in a single unsteady inviscid computation.

The numerical procedure is briefly described in Section 2 together with the boundary conditions formulation. In Section 3, the unsteady results are presented. The identification of the different phases of the basic stator trailing edge shock wave impinging on the moving downstream rotor is performed in Section 3.1. The migration of the hot streak as it is chopped by the moving rotor blades is discussed in Section 3.2. The interaction between the shock waves and the hot streaks is treated in Section 3.3. Although the unsteady solution is necessary to get the knowledge of the maximum temperature, the time-averaged flow solution is important as the turbine designer has to rely on mean flow values. Hence, the time-average of the periodic unsteady solution is presented in Section 4. This allows to assess the mean effect of the shock and the streak unsteadiness. The comparison of the time-averaged and the steady-state solutions on the rotor blade surfaces allows to assess the importance of the circumferential streak variations (not retained in the steady-state mode) on the rotor flow field. A summary and the essential conclusions are given in Section 5.

2 NUMERICAL PROCEDURE

The time-dependent Euler equations are solved using a node-based, explicit Ni-Lax-Wendroff scheme (Ni, 1981). The extension of this procedure to 3-D on unstructured meshes with hexahedral cells has been reported previously by the first author (Saxer, 1992). The mesh is found by solving Poisson equations with source-terms calibrated to control spacing and orthogonality at the blade/hub/tip surfaces. A combined second and fourth-difference numerical smoothing acting on the state-vector is added to the algorithm to prevent high frequency oscillations in the solution as well as to capture shocks. The fourth-difference smoothing, an extension to 3-D of the method of Holmes and Connell (1989), exploits the advantages of a pseudo-Laplacian to ensure second-order accuracy in shock-free regions even on distorted grids. This is a desired feature to study inlet distortion effects or to compare different solutions without spurious distortions due to numerical smoothing. A non-linear second-difference operator is used to capture shocks with an artificial bulk viscosity parameter tailored by the local flow divergence and the Mach number. This avoids large shock overshoots, and does not alter the global accuracy of the scheme in smooth flow regions. In each blade row, relative flow variables are used.

2.1 Boundary Conditions

Numerical and physical boundary conditions have to be applied at the stator inlet, stator/rotor interface and rotor exit as well as on solid and periodic surfaces. The stator and most of the rotor flow is choked at the throat, indicating that the periodic unsteadiness created by the relative motion between the stator and the rotor blades does not reach the rotor exit and the stator inlet. Hence, steady-state boundary conditions are applied at both the stator inlet and the rotor exit. At the stator inlet, the 1-D characteristics theory is used to prescribe the stagnation enthalpy, the radial and tangential flow angles as well as the entropy (or stagnation pressure). At the rotor exit, the quasi-3-D non-reflecting boundary conditions formulation developed by Saxer and Giles (1991) is used together with the requirement that the outflow is in radial equilibrium. This avoids spurious reflections of the rotor trailing edge shock wave off the exit of the computational domain.

For time-accurate calculations of stator/rotor flow fields the changes required by the Lax-Wendroff algorithm from a time level to the next one at the stator outflow and at the rotor inflow are set to eliminate the following characteristic jumps taking note of the direction of propagation of each characteristic:

\[
\begin{pmatrix}
\Delta \phi_1 \\
\Delta \phi_2 \\
\Delta \phi_3 \\
\Delta \phi_4
\end{pmatrix} =
\begin{pmatrix}
-\epsilon^2 & 0 & 0 & 0 \\
0 & \rho c & 0 & 0 \\
0 & 0 & \rho c & 0 \\
0 & 0 & 0 & \rho c
\end{pmatrix}
\begin{pmatrix}
\rho & -\rho c \\
\rho u_x & -u_x \\
\rho u_y & -u_y \\
\rho u_z & -u_z
\end{pmatrix}
-\Omega R
\]

where \( \rho, u_x, u_y, u_z \) and \( p \) represent the values of density, axial, circumferential and radial velocity components and pressure, respectively. Note that because of the use of relative flow variables, the rotor wheel speed \( \Omega R \) has to be introduced into the condition of matching circumferential velocities. The outgoing characteristics changes are calculated by the Lax-Wendroff algorithm. The combined five characteristic changes on both side of the interface are then converted back to primitive and finally conservation variables before the flow field is updated.

In a steady-state calculation of a stator/rotor interaction, the quasi-3-D non-reflecting boundary conditions mentioned above are used together with a circumferential stream-thrust flux-averaging technique in order to conserve mass, momentum and energy across the mixing plane between a stator and a rotor row. Hence, radial variations are automatically accounted for and the stator and the rotor flow fields are matched at the interface (Saxer, 1992).

In addition to these boundary conditions, a no mass flux condition is enforced at the hub and the tip endwalls as well as on the stator and the rotor blades. Also, periodic boundary conditions along the outer boundaries of the H-type grid are enforced in the tangential direction.

3 UNSTEADY SOLUTION

The numerical procedure discussed above is applied to the computation of the unsteady shock wave/hot streak interaction occurring in the transonic first turbine stage represented in Figure 1. The design of this stage with a 4:1 pressure ratio was performed by Rolls-Royce as part of the Advanced Core Engine program (ACE). It is representative of a high pressure, cooled aircraft turbine operating in the transonic regime. The flow simulation uses a 3-to-5 stator-to-rotor blade count ratio and periodic boundary conditions, which corresponds very closely to the actual configuration of (36 stators)/(61 rotors). 56 x 36 x 21 nodes are used in each of the three stator channels, and 56 x 22 x 21 in each of the five rotor passages. Starting from the steady-state solution, the unsteady computation took 12 stator periods (or 20 rotor periods) to converge to a periodic solution, using 1300 iterations/cycle. A CPU
time of $241 \times 10^{-6}$ seconds/iteration/grid point was required on a Silicon Graphics Indigo R4000 workstation.

The stator inlet conditions are set by specifying a radial and a tangential flow angle, a stagnation pressure, and a total temperature distribution in the form of a streak of hot gas. The streak is placed midway between Stator S1 and Stator S2 at 60% span, see Figures 3 and 9 (top, left). Due to periodicity this means that a hot streak is introduced in one out of every three stator passages. Relative to the surrounding freestream, the peak stagnation temperature ratio is 1.97. The total pressure is assumed to be uniform over the entire inlet section, which means that the streak-to-freestream stagnation temperature ratio is the reciprocal of the streak-to-freestream total density ratio. The assumption of constancy of the inlet total pressure is motivated by the low Mach number at the exit of the combustion chamber considered as a plenum, and the neglect of viscous effects.

The transonic nature of the flow field and the introduction of the hot streak produce two distinct unsteady stator/rotor interaction phenomena. For supersonic vane exit conditions, a system of oblique shock waves is generated at the trailing edge of the stator. For small axial gaps, the suction side trailing edge shock extends to the downstream rotor and impinges on the suction side. This produces reflected waves on both the adjacent rotor and on the upstream stator causing unsteady blade loading. This is discussed in Section 3.1. The unsteady temperature migration in the rotor passages due to the periodic chopping of the streak of hot gas exiting the vane by the rotors is analyzed in Section 3.2. The unsteady shock wave/hot streak interaction is discussed in Section 3.3.

### 3.1 Unsteady Shock Motion

Figure 2 shows the instantaneous static pressure contours at a constant radius set at 60% span (thus corresponding to a cut through the core of the streak at the inlet), at eight intervals during one periodic cycle. Note that a cycle, starting at $t = 0$, is defined here as the time for one rotor passage to rotate by an amount of five rotor pitches to $t = 1$, which by periodicity also corresponds to $t = 0$. As opposed to the three stator period for the hot streak migration, the stator/rotor shock interaction occurs during one stator period. Hence, a complete view of the unsteady shock motion in one rotor passage would have required to store many more time-dependent solutions. However, this can be circumvented by noting that a basic phenomenon occurring in a given rotor passage at a given instant can also be seen in another passage at another time. This implicitly assumes that the hot streak has only a weak effect on the basic stator/rotor shock mechanism and that only the relative position between stator and rotor is of importance. As the results will show, this seems to be a valid assumption.

Examining the pressure contours at the beginning of the cycle at $t = 0$, at the bottom of the figure the oblique shock extending downstream from the trailing edge of Stator S1 is just about to impact on the suction side of Rotor R1. The actual impact on the crown of the rotor is seen at time $t = 0.750$ between Stator S2 and Rotor R4, where also a weak reflection is noticeable. As the rotor blade moves upward, the location of the shock wave impingement moves forward towards the leading edge, see for example Rotor R1 at $t = 0.375$. As this occurs, the reflection is also growing stronger, see Rotor R4 at $t = 0.125$.

At $t = 0.250$ for Rotor R2, the primary reflection point has reached the rotor leading edge, and a primary reflected wave is clearly visible. The leading edge shock wave impact is also noticeable for Rotor R3 at $t = 0.375$ and R5 at $t = 0.625$. In addition to this primary reflection, the portion of the reflected wave which moves towards the pressure surface of the adjacent rotor is reflecting a second time and is moving back towards the original rotor, see R5 at $t = 0$.

As the rotor continues to move, the primary reflection passes the leading edge, see R2 at $t = 0.250$ for example. From now on the stator oblique shock no longer impacts on the rotor. The curved diffracting part of the shock wave (see for example R5 at $t = 0.000$ and R4 at $t = 0.875$) is weak and is continuing to weaken. However, the length of its straight portion is increased (due to a velocity component approximately $\sqrt{V^2 - c^2}$ along the shock front). Compare for example the stator trailing edge shock wave S1 at $t = 0.750$ with S3 at $t = 0.250$.

While the original stator shock no longer impacts on the rotor surface, the primary reflected wave has left the rotor and is propagating upstream towards the stator suction surface, see interaction S2 with R4 at $t = 0.875$, and the middle of the figure at $t = 0.625$ (bowed pressure contours between R2, R3 and S1). Also seen in the passage between Rotor R4 and R5 is the crossing back of the secondary reflected shock from the pressure side of R4 towards the suction side of R5. As it crosses back from a low Mach number to a larger Mach number region, towards the original rotor, the secondary reflection gets stronger, see for example R4 at $t = 0.875$, R2 at $t = 0.625$ and R1 at $t = 0.500$.

At $t = 0.875$ the primary reflected wave from Rotor R1 has just struck the suction side of Stator S1 in a region close to the trailing edge, and moves back towards the adjacent rotor. However, this reflection is not much visible on the contour plots. As the rotor moves away from the stator primary oblique shock, the latter regains its maximum strength, see S2 at $t = 0.125$, until it strikes the next rotor, and the basic shock mechanism starts again.

For the case examined here, the unsteady stator/rotor shock interaction is essentially a two-dimensional process driven by the oblique shock leaving the stator trailing edge and impinging on the moving rotor blade. This unsteady shock interaction is similar in nature to the result presented by Giles (1990a) in a 2-D time-accurate numerical simulation which included quasi-3-D source terms. However, some differences appear due to the hot streak interaction and 3-D effects.

### 3.2 Unsteady Temperature Migration

The objective of this section is to analyze the migration of the hot streak introduced at the inlet of the transonic turbine stage of Figure 1. Figure 3, showing the time-dependent stagnation temperature contours at 60% span is the pendant of Figure 2. A uniform flow of same mass-averaged stagnation temperature as the one impo-

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side part of the leading edge, the streak width tightens in the interblade channel. This effect is clearly seen at all times on the suction surface of two consecutive blades. The V-shape in the blade passage, due to the acceleration gradient, is accentuated as the streak convects down the stream of this area, the unsteady pressure waves resulting from the stator/rotor shock and potential interaction discussed in Section 3.1 reach the aft part of the stator suction surface. This affects the streak path and width.

Due to the absence of secondary flow, no significant temperature redistribution occurs in the stator during the entire cycle. At the stator inlet in the streak region, the boundary conditions produce a radial as well as a tangential sheared velocity field, hence vorticity. However, under the invariance of the inlet stagnation pressure and by using the Munk and Prim substitution principle (Munk and Prim, 1947), it can be shown that no 3-D secondary flow associated with the temperature gradient occurs in the vane (Saxer, 1992). Indeed, the Munk and Prim principle states that if a steady isentropic flow field without body forces is determined for a specific geometry and a total pressure distribution, then the streamlines pattern, Mach number and the static pressure fields remain unchanged for any other stagnation temperature distribution. Another way of checking this statement is to use secondary flow theory (Hawthorne, 1974, Johnson, 1978, Horlock and Lakshminarayana, 1973). It can be shown that in the absolute frame of reference the growth of the absolute streamwise vorticity is dependent upon the gradient of stagnation pressure, which is set to zero at the inlet, i.e. the vortex lines introduced at the inlet have to remain perpendicular to the flow in the vane. In fact, the secondary flow developed by the turning of the inlet normal vorticity is exactly balanced by the secondary flow introduced by the temperature gradient. These arguments are valid up to the vane choked throat. Downstream of this area, the unsteady pressure waves resulting from the stator/rotor shock and potential interaction discussed in Section 3.1 reach the aft part of the stator suction surface. This affects the streak path and width.

Examining the interaction between the rotor blade and the streak during one cycle, five different phases of the streak redistribution can be identified. The first phase consists of the hot streak crossing the interface and being convected towards the blade row (t = 0, middle of the figure). The second phase is when the hot streak impinges on the leading edge and starts to separate into the suction and pressure side legs, as seen for R3 at t = 0, R2 at t = 0.125 or R4 at t = 0.750. In the stagnation point region on the pressure side hot fluid seems to attach to the blade, while cooler surrounding gas wraps around it forming a 'socket' of hot fluid slowly convecting along the pressure surface, see for example R2 at t = 0.250, R3 at t = 0, and R1 at t = 0.500. Meanwhile on the suction side part of the leading edge, the streak width tightens under the local acceleration of the flow.

The next phase is the bowing of the streak into a V-shape in the blade passage, due to the acceleration gradient in the high speed region between the pressure and the suction surface of two consecutive blades. The V-shape distortion is accentuated as the streak converges down the interblade channel. This effect is clearly seen at all times for different blade passages.

Phase four is the merging of the pressure and suction side legs of the streak in the second part of the blade channel and in the exit region. Finally, the remainder of the chopped streak convects outside of the domain.

Figure 4 represents the rotor-relative stagnation temperature distribution on eleven axial cuts at time t = 0 during the streak/rotor interaction. This allows to identify the extent of the streak radial migration, as well as the mixing of the suction and the pressure side leg of the hot gas in the five computed rotor passages. As in Figure 3, the lower and upper temperature levels have been filtered out to better see the streak migration.

The unsteady rotor-relative stagnation temperature contours and static pressure contours on the pressure surface of Rotor R3 are plotted in Figure 5. Figure 6 shows these same quantities on the suction surface of Rotor R3. Clearly the pattern of the streak migration from the leading edge towards the trailing edge is different on these two surfaces, with a significant secondary flow in the radial direction on the pressure surface. This forces hot midspan fluid towards the hub and the tip endwalls. Also, the 'residence' time of the hot streak is smaller on the suction surface than on the pressure side, due to the generally lower convection speed on the pressure side. On the rotor pressure surface the streak 'residence' time corresponds approximately to 1/2 of a cycle period, i.e. roughly 1.5 rotor periods, whereas only two rotor periods are needed for the streak to transit on the suction surface.

3.3 Unsteady Shock Wave/Hot Streak Interaction

The unsteady pressure waves distribution discussed in Section 3.1 will influence both the path and the strength of the hot streak as it migrates in the rotor row. Fluid particles crossing a compression wave, such as the one attaching from the rotor leading edge (primary reflection) and moving back towards the stator will have their trajectories slightly deviated, see for example Figures 2 and 3 at t = 0.750. Also, according to the energy equation written in the rotating frame of reference,

\[
\frac{D}{Dt} = \frac{1}{\rho} \frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} \right),
\]

the shock wave increases the local rotor-relative stagnation enthalpy, hence total temperature. In Equation (2) \(D/Dt\) represents the material derivative, whereas \(I = (h_t) - 0.5(\Omega R)^2\) is the rotational term. For example, this effect can be seen on the suction surface as the stator trailing edge shock wave impinges on the blade (R3 at t = 0, see Figs. 2, 3 and 6). Also, as the secondary reflection off the adjacent row strengthens into a shock near the rotor crown (R3 at t = 0.125, see Figs. 2, 3 and 6), it increases the local blade surface temperature. By recording the streak peak stagnation temperature at the rotor inlet during one cycle, one can estimate the effect of the shock waves interaction on the temperature distribution. Compared to the rotor inlet time-averaged maximum value (i.e. time-averaged value of the streak core), variations as much as 12% are observed.

Figures 7 and 8 present the static pressure and the relative stagnation temperature distributions on the rotor blade surface, respectively. The difference between...
the maxima and the minima values on both the pressure and the suction sides, i.e. the envelopes of unsteadiness, provide an estimate of the overall effect of unsteadiness during one cycle. These values were extracted from eight unsteady snapshots during the final blade-passing cycle. Thus, a certain amount of discreetness is observed in those quantities. Notice, however, that the time-averaged solution also presented in Figures 7 and 8 have been computed during the blade-passing period from all the time-steps. Examining the static pressure envelopes of Fig. 7 at three spanwise locations one notices that unsteadiness is particularly intense on the forward portion of the blade due to the primary reflection off the suction surface, with peak-to-peak variations in the order of half the (stator) inlet reference stagnation pressure. The level of unsteadiness is generally lower on the pressure side than on the suction side, except near the area of impact of the secondary shock reflection on the pressure side in the second portion of the blade.

As seen on Figure 8, the unsteady rotor-relative stagnation temperature fluctuations are larger at 60% span than at the hub and the tip, which is a clear indication of the transit of the hot streak. On the pressure side, these midspan variations account for more than half the (stator) inlet reference total temperature. Also, at 60% span the maxima in $T_r$ are larger on the pressure side than on the suction side. The effect of the streak pressure side radial migration is visible through the increase in total temperature near the trailing edge at the tip. Since the streak doesn’t quite extend to the root of the blade (recall Figs. 5 and 6), the variation in relative stagnation temperature in the absence of streak transit can be assessed by considering the root unsteady total temperature envelope. For the case presented here, this accounts for 15% $T_i$, which is 3.5 times less than at midspan (streak crossing).

On the stator blade surface, the unsteady (absolute) total temperature variation accounts for 10% of the inlet distortion, whereas the unsteady static pressure envelope represents about 25% of $p_{t,\text{in}}$ (due to choked flow conditions, the unsteadiness is restricted to the end portion of the suction side).

4 TIME-AVERAGED AND STEADY-STATE RESULTS

As mentioned in the introductory section, the turbine blade designer has to rely upon mean flow quantities. The objective of this section is to assess the mean effect of the shock and the streak unsteadiness in the first turbine stage. The flow conditions for the steady-state and for the time-averaged periodic unsteady computations are listed in Table 1. To provide a basis for comparison, the steady-state computation was performed with uniform inlet conditions (i.e. $p_{t,\text{in}} = \text{cst}$, $T_{t,\text{in}} = \text{cst}$, no streak), but with identical mass flow. On a time-averaged basis, the crossing of the hot streak produces an increase in the rotor blade surface total temperature near the rotor exit. The position of the cuts is marked at the top of Figure 10. Absolute and relative stagnation temperatures are plotted in the stator and the rotor, respectively. The decrease of the peak temperature from the stator inlet to the stator exit is an indication of the numerical dissipation acting upon the streak core supported by too few grid nodes.

From a time-averaged point of view the ‘one hot streak in three stator passages’ has been transformed into a ‘one lukewarm streak in each of the five rotor channels’, with a reduced core peak temperature level. For example, the maximum unsteady non-dimensionalized inlet rotor total temperature is about 1.58, which compares to 0.92 in the time-averaged solution. The ratio of these numbers corresponds approximately to the ratio of stator-to-rotor pitch. On a time-averaged basis in the rotor, the suction side core of the streak retains its original shape but migrates slowly towards the pressure side. As opposed to the pressure surface, the maximum temperature stays off the blade suction surface.

In Figure 10, the time-averaged stagnation temperature and static pressure are represented on a cut at 60% span. An interesting feature to notice is that the maximum temperature in the rotor domain occurs in the field near the leading edge, and not on the blade itself or at the rotor inlet.

The time-averaged rotor-relative stagnation temperature is larger on the pressure surface than on the suction side, see Figure 11. Also, the areas of high temperatures cover a broader portion of the pressure surface than on the suction surface. This is a consequence of the radial secondary flow occurring on the pressure side and mentioned in Section 3.2. This effect can be qualitatively explained by using vorticity and velocity triangles arguments and simple dynamics (Butler et al., 1986, Saxer, 1992). Figure 12 is a schematic of the vortex and the velocity triangles at the stator/rotor interface. In the relative frame, the stator vorticity (normal to the absolute flow) can be decomposed into a normal and a streamwise component, which will strengthen as the normal component turns into the rotor passage producing a 3-D secondary flow. From the velocity triangle it can be inferred that in the rotor frame the hot core fluid is streaming with a relative higher velocity than the cold surrounding fluid, and with a larger incidence angle. The hot gas has a ‘slip velocity’ component normal to the surrounding cooler fluid that drives it towards the pressure surface. The overall effect is the collection of hotter gas on the rotor pressure side than on the suction side. On the pressure side, the hot fluid is spreading from midspan towards the hub and the tip endwalls resulting in the heating of the upper and lower walls. On the suction side, cool endwall gas flows towards midspan. Under the influence of the secondary flow, as schematized in Figure 13, the general motion of the hot core moving across the passage towards the rotor pressure surface can be seen in Figure 9. Notice that the time-averaged surface total temperature downstream of the leading edge on both the pressure and the suction sides is higher than at the leading edge itself.

Figure 14 represents the steady-state stagnation temperature and static pressure distributions on the rotor pressure and suction surfaces. This computation was performed using the steady-state non-reflecting boun-
dary conditions and flux-averaging procedure mentioned in Section 2.1, where the tangential components of the hot streak at the exit of the stator have been flux-averaged, retaining only its radial variations in the rotor frame. By comparing Fig. 14 with Fig. 11 one infers that, regarding the pressure distribution, the steady-state procedure is in very good agreement with the time-averaged of the periodic unsteady solution. However, the radial extent of the temperature distortion is much weaker than in the unsteady case. This suggests that the streak tangential variations, that have been retained in the rotor time-averaged solution are of prime importance in the generation of the radial secondary flow on the blade pressure surface.

5 SUMMARY AND CONCLUSIONS

This paper has presented a numerical simulation of the unsteady inviscid and three-dimensional flow in a generic industrial transonic first turbine stage in the presence of streaks of hot gas. Realistic flow conditions were obtained by setting a peak-to-free-stream temperature ratio of 1.97 at the inlet of one out of every 3 stator passages and by using a 3-to-5 stator-to-rotor blade count ratio.

Due to the impact of the stator trailing edge shock wave off the downstream rotor, a shock motion was captured as part of the three-dimensional solution. The unsteady shock interaction included a primary reflection off the rotor suction surface and a secondary reflection off the adjacent rotor pressure side. The primary reflection moves towards the upstream stator striking it on the suction side close to the trailing edge. In addition, the secondary reflection crosses back towards the original rotor and intensifies into a discernable shock. This shock motion is consistent with previous 2-D numerical simulations. The computed unsteady static pressure fluctuations are the largest on the forward portion of the rotor suction surface and can exceed half of the stator inlet stagnation pressure.

Due to the invariance of the inlet stagnation pressure, the hot streak is convected without redistribution in the stator passage. The different phases of the hot streak chopping and migration in the rotor frame were presented and discussed. In particular, the presence of a secondary flow force the chopped pressure side leg of the hot streak to migrate towards the hub and the tip endwalls. The unsteadiness produced by the transonic interaction affects the streak peak stagnation temperature at the rotor inlet by more than 10% of the stator inlet temperature distortion. As an estimate, the streak crossing time from the rotor leading edge to the trailing edge corresponds approximately to 4 rotor periods on the pressure surface and 2 rotor periods on the suction side. As a consequence, the time-averaged rotor surface stagnation temperature is larger on the pressure side than on the suction side, but surprisingly only to a small extent. It is suggested that the increase in the local rotor-relative stagnation temperature is produced by the unsteady shock wave interaction particularly active on the forward portion of the suction side area, thus increasing the level of the time-averaged value to nearly the one obtained on the pressure surface.

On a time-averaged basis, each rotor passage experiences the effects of an inlet hot streak of circular shape and reduced core temperature with respect to the streak temperature at the exit of the stator. The ratio of time-averaged maximum streak temperatures (exit stator)/(inlet rotor) scales approximately like the stator-to-rotor count ratio. Due to the secondary flow, the maximum time-averaged temperature on the rotor blade occurs on the pressure surface in a region centered at 10% rotor inlet span above the rotor inlet core position.

A comparison between the time-averaged solution and the steady-state solution, in which the circumferential variations introduced by the streak are cut off (but not the radial variations), was performed. The pressure field compares very well between the two solutions, but not the stagnation temperature distribution. This suggests that the tangential temperature distortions at the rotor inlet have a stronger effect on the radial secondary flow than the streak radial variations. Hence, accurate predictions of temperature distributions in the presence of streaks of hot gas requires any numerical procedure to account for unsteadiness as well as three-dimensional effects.

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REFERENCES


<table>
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<th>parameter</th>
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<td>Stator inlet tangential angle</td>
<td>hub 0.0°, 0.0°, 0.0°</td>
<td>hub 0.0°, 0.0°, 0.0°</td>
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<tr>
<td></td>
<td>60% span 0.0°, 0.0°</td>
<td>60% span 0.0°, 0.0°</td>
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<tr>
<td></td>
<td>tip 0.0°, 0.0°, 0.0°</td>
<td>tip 0.0°, 0.0°, 0.0°</td>
</tr>
<tr>
<td>Stator outlet tangential angle</td>
<td>hub 74.5°, 74.5°, 74.5°</td>
<td>hub 74.5°, 74.5°, 74.5°</td>
</tr>
<tr>
<td></td>
<td>60% span 75.0°, 75.0°</td>
<td>60% span 75.0°, 75.0°</td>
</tr>
<tr>
<td></td>
<td>tip 75.0°, 75.0°, 75.0°</td>
<td>tip 75.0°, 75.0°, 75.0°</td>
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<tr>
<td>Rotor-relative inlet tangential angle</td>
<td>hub 55.0°, 55.0°, 55.0°</td>
<td>hub 55.0°, 55.0°, 55.0°</td>
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<tr>
<td></td>
<td>60% span 48.7°, 48.7°</td>
<td>60% span 48.7°, 48.7°</td>
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<td>tip 38.0°, 38.0°, 38.0°</td>
<td>tip 38.0°, 38.0°, 38.0°</td>
</tr>
<tr>
<td>Rotor-relative outlet tangential angle</td>
<td>hub -59.1°, -59.1°, -59.1°</td>
<td>hub -59.1°, -59.1°, -59.1°</td>
</tr>
<tr>
<td></td>
<td>60% span -70.1°, -70.1°</td>
<td>60% span -70.1°, -70.1°</td>
</tr>
<tr>
<td></td>
<td>tip -70.1°, -70.1°, -70.1°</td>
<td>tip -70.1°, -70.1°, -70.1°</td>
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<tr>
<td>Stator inlet Mach</td>
<td>hub 0.13, 0.13, 0.13</td>
<td>hub 0.13, 0.13, 0.13</td>
</tr>
<tr>
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<td>60% span 1.31, 1.31, 1.31</td>
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<td></td>
<td>tip 1.22, 1.22, 1.22</td>
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<tr>
<td>Stator outlet Mach</td>
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<td>hub 1.03, 1.03, 1.03</td>
</tr>
<tr>
<td></td>
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<td>60% span 1.10, 1.10, 1.10</td>
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<tr>
<td></td>
<td>tip 1.07, 1.07, 1.07</td>
<td>tip 1.07, 1.07, 1.07</td>
</tr>
<tr>
<td>Rotor-Mach</td>
<td>hub 0.76, 0.76, 0.76</td>
<td>hub 0.76, 0.76, 0.76</td>
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<tr>
<td></td>
<td>60% span 0.80, 0.80, 0.80</td>
<td>60% span 0.80, 0.80, 0.80</td>
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<tr>
<td></td>
<td>tip 0.87, 0.87, 0.87</td>
<td>tip 0.87, 0.87, 0.87</td>
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<tr>
<td>Rotor wheel speed [G/(slug/ft³)]</td>
<td>1.37, 1.37, 1.37</td>
<td>1.37, 1.37, 1.37</td>
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<tr>
<td>Ratio of specific heats γ</td>
<td>1.27, 1.27, 1.27</td>
<td>1.27, 1.27, 1.27</td>
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Table 1: Steady and time-averaged flow parameters for transonic turbine stage (same mass-flow in both cases).
Figure 2: Unsteady static pressure contours at 60% span ($\Delta p/p_{\infty} = 0.02$).

Figure 3: Unsteady stagnation temperature contours at 60% span ($\Delta T_{i}/T_{i,\infty} = 0.03$, except in streak core) (stator: absolute $T_{i}/T_{i,\infty}$, rotor: relative $T_{i}/T_{i,\infty}$).
Figure 4: Rotor-relative stagnation temperature contours on axial cuts at time $t=0.0$, ($\Delta T_r/T_{\infty} = 0.03$, ss: suction side, ps: pressure side).
Figure 5: Unsteady rotor-relative stagnation temperature (top) and static pressure (bottom) on pressure surface of rotor R3 (see Figs. 2 and 3) \((\Delta T_r/T_{r_in} = 0.03, \Delta p/p_{r_in} = 0.02)\).

Figure 6: Unsteady rotor-relative stagnation temperature (top) and static pressure (bottom) on suction surface of rotor R3 (see Figs. 2 and 3) \((\Delta T_r/T_{r_in} = 0.03, \Delta p/p_{r_in} = 0.02)\).
Figure 7: Time-averaged and unsteady static pressure on rotor blade.

Figure 8: Time-averaged and unsteady rotor-relative stagnation temperature on rotor blade.

Figure 9: Stagnation temperature contours on axial cuts for the time-averaged solution ($\Delta T_i/T_{\infty} = 0.01$, for $x/c=-1.93 \& -0.30$ $\Delta T_i/T_{\infty} = 0.06$) (ss: suction side, ps: pressure side).
Figure 10: Stagnation temperature and static pressure contours at 60% span for the time-averaged solution ($\Delta T_i/T_{i\infty} = 0.01, \Delta p/p_{i\infty} = 0.02$).

Figure 11: Rotor-relative stagnation temperature and static pressure for the time-averaged solution ($\Delta T_i/T_{i\infty} = 0.01, \Delta p/p_{i\infty} = 0.02$).

Figure 12: Velocity and vorticity triangles at stator/rotor interface.

Figure 13: Schematic of rotor-relative secondary flow for the time-averaged solution (ss: suction side, ps: pressure side).

Figure 14: Rotor-relative stagnation temperature and static pressure for the steady-state solution ($\Delta T_i/T_{i\infty} = 0.01, \Delta p/p_{i\infty} = 0.02$).